Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/95-4.2.7-d-trig-^m-a+b-c-cos-^n-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [98]. This is test number [95].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (98)	0.00 (0)
Mathematica	100.00 (98)	0.00 (0)
Maple	100.00 (98)	0.00 (0)
Fricas	82.65 (81)	17.35 (17)
Giac	77.55 (76)	22.45 (22)
Maxima	71.43 (70)	28.57 (28)
Mupad	68.37 (67)	31.63 (31)
Sympy	19.39 (19)	80.61 (79)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

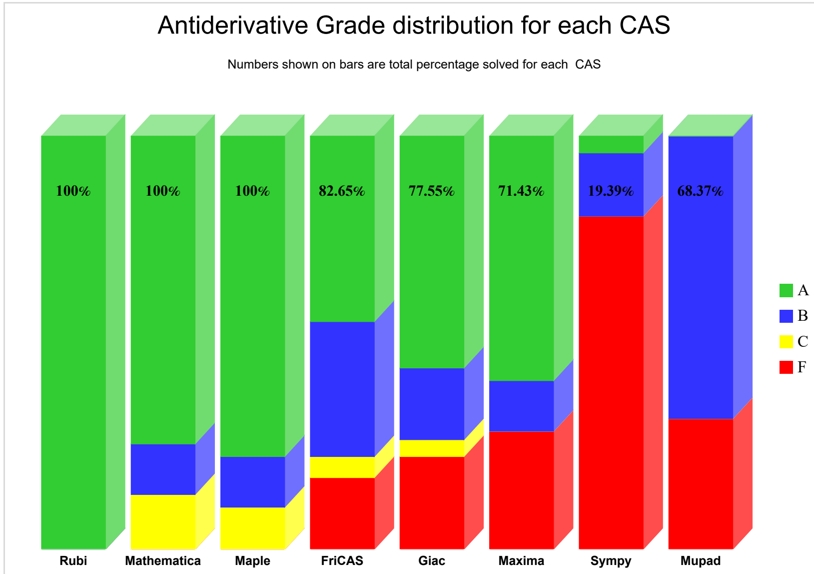
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

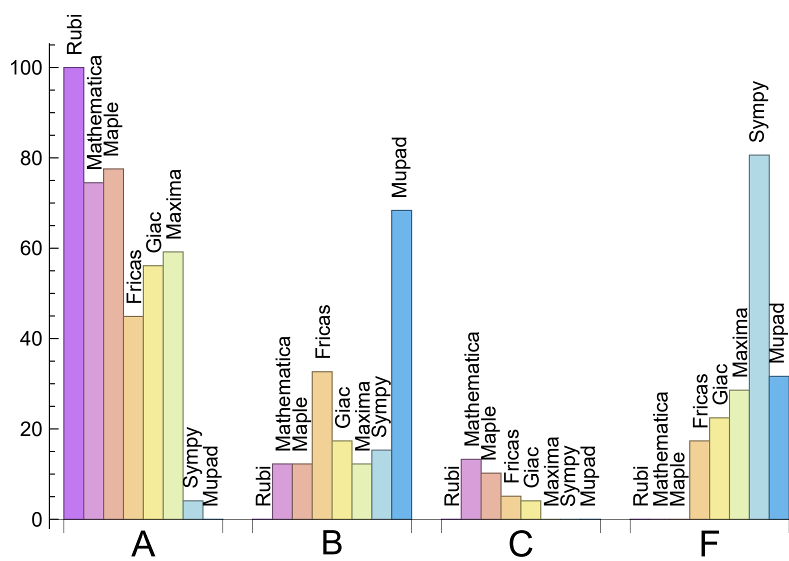
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	77.55	12.24	10.20	0.00
Mathematica	74.49	12.24	13.27	0.00
Maxima	59.18	12.24	0.00	28.57
Giac	56.12	17.35	4.08	22.45
Fricas	44.90	32.65	5.10	17.35
Sympy	4.08	15.31	0.00	80.61
Mupad	N/A	68.37	0.00	31.63

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	17	47.06 %	23.53 %	29.41 %
Giac	22	90.91 %	9.09 %	0.00 %
Maxima	28	100.00 %	0.00 %	0.00 %
Sympy	79	72.15 %	27.85 %	0.00 %
Mupad	31	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

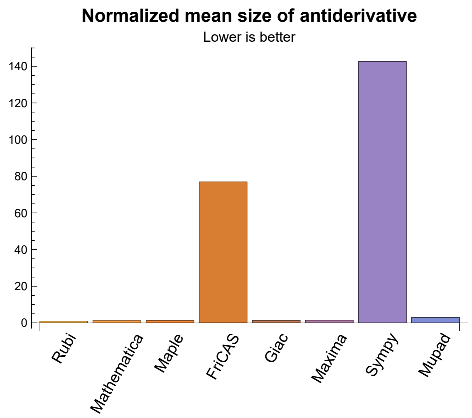
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

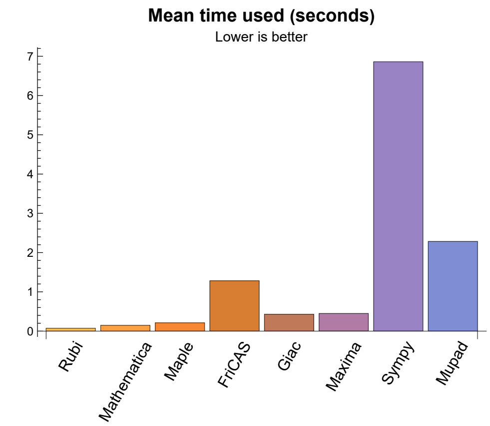
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	71.99	1.00	41.50	1.00
Mathematica	0.15	68.32	1.20	45.50	1.00
Maple	0.21	68.73	1.23	48.50	0.97
Maxima	0.45	58.53	1.49	41.50	1.09
Fricas	1.28	16824.02	76.92	119.00	2.90
Sympy	6.86	4203.79	142.58	78.00	3.50
Giac	0.43	68.03	1.43	46.50	1.24
Mupad	2.28	274.70	3.00	75.00	1.06

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
20
40
60
80
100
120
140
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{74, 76, 77, 79\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

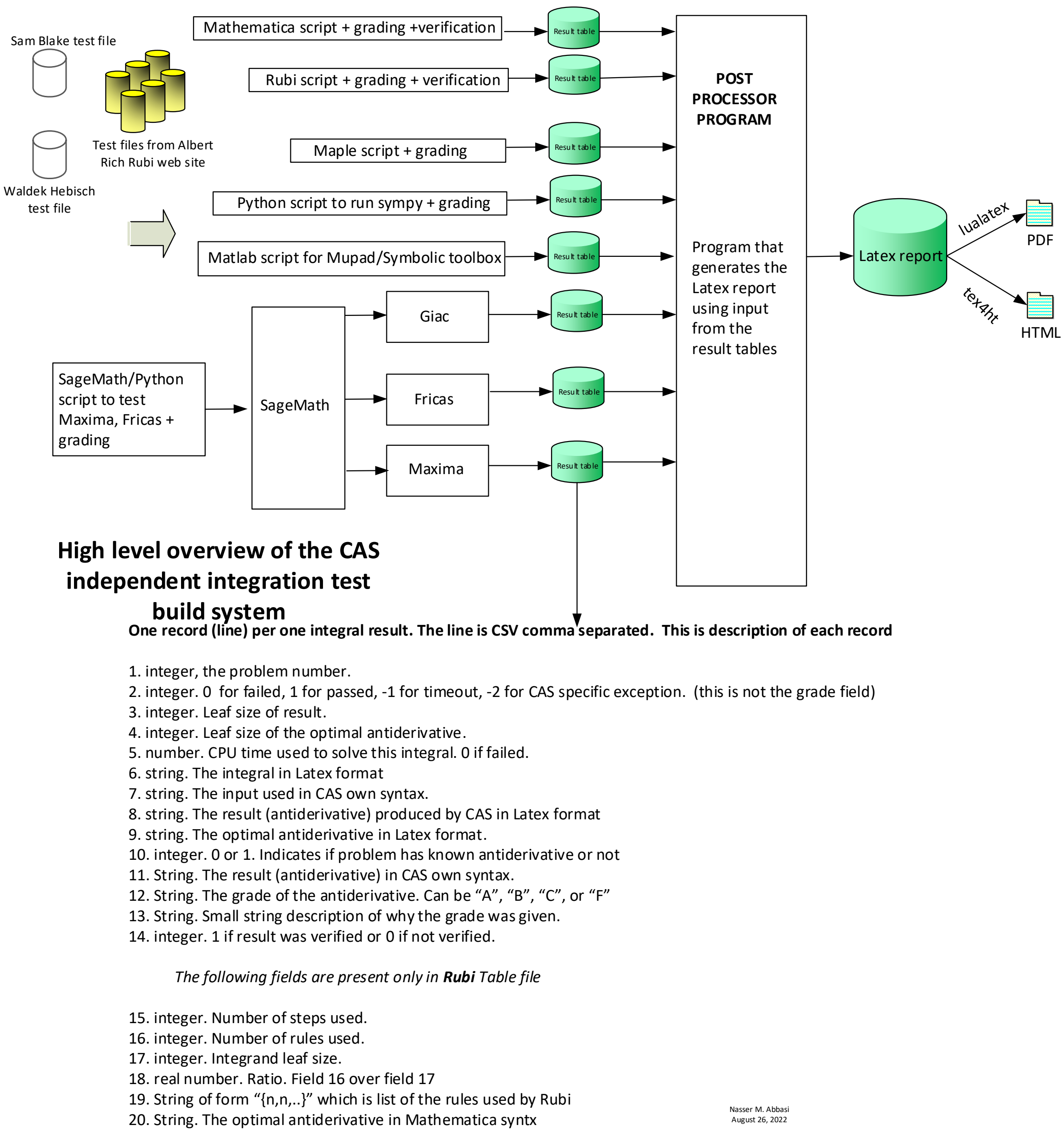
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 10, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 73, 81, 85, 86, 87, 89, 90, 93, 94, 95, 96, 97, 98 }

B grade: { 6, 7, 9, 11, 12, 15, 16, 32, 33, 34, 88, 91 }

C grade: { 70, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 92 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 57, 59, 60, 62, 63, 65, 66, 71, 72, 73, 81, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 52, 55, 56, 58, 61, 64, 67, 68, 69, 70, 84, 85 }

C grade: { 5, 74, 75, 76, 77, 78, 79, 80, 82, 83 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 67, 68, 69, 73, 81, 86, 92, 93, 94, 95, 97, 98 }

B grade: { 6, 7, 15, 16, 51, 53, 54, 87, 88, 89, 90, 91 }

C grade: { }

F grade: { 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 96 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 17, 18, 19, 24, 25, 28, 29, 30, 32, 33, 34, 35, 36, 37, 44, 45, 46, 47, 49, 51, 53, 62, 73, 86, 87, 88, 89, 90, 93, 95, 96, 97, 98 }

B grade: { 6, 7, 9, 15, 16, 20, 21, 22, 23, 26, 27, 31, 38, 39, 40, 41, 42, 43, 61, 64, 65, 67, 68, 69, 70, 71, 72, 76, 79, 81, 85, 91 }

C grade: { 63, 66, 75, 78, 92 }

F grade: { 48, 50, 52, 54, 55, 56, 57, 58, 59, 60, 74, 77, 80, 82, 83, 84, 94 }

2.1.6 Sympy

A grade: { 5, 24, 44, 73 }

B grade: { 1, 2, 3, 4, 6, 13, 20, 25, 26, 27, 31, 38, 45, 46, 84 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 21, 22, 23, 28, 29, 30, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 70, 72, 73, 86, 87, 89, 92, 93, 94, 95, 96, 97, 98 }

B grade: { 6, 7, 15, 16, 23, 47, 49, 53, 67, 69, 71, 81, 84, 85, 88, 90, 91 }

C grade: { 48, 50, 52, 54 }

F grade: { 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 74, 75, 76, 77, 78, 79, 80, 82, 83 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 55, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92 }

C grade: { }

F grade: { 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	31	37	23	473	31	25
N.S.	1	1.00	0.79	0.94	1.12	0.70	14.33	0.94	0.76
time (sec)	N/A	0.034	0.006	0.092	0.485	0.382	2.257	0.414	2.114

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	14	14	78	14	16
N.S.	1	1.00	1.00	0.84	0.74	0.74	4.11	0.74	0.84
time (sec)	N/A	0.031	0.005	0.067	0.271	0.379	1.365	0.422	2.035

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	23	21	14	153	22	15
N.S.	1	1.00	0.90	1.15	1.05	0.70	7.65	1.10	0.75
time (sec)	N/A	0.029	0.005	0.064	0.478	0.380	0.811	0.426	2.037

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	12	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.71	1.00	1.00
time (sec)	N/A	0.025	0.004	0.053	0.262	0.379	0.463	0.406	0.024

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	2	5	5
N.S.	1	1.00	1.00	1.60	1.00	1.00	0.40	1.00	1.00
time (sec)	N/A	0.025	0.001	0.059	0.476	0.381	0.257	0.421	2.050

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	21	9	21	22	19	23	8
N.S.	1	1.00	2.62	1.12	2.62	2.75	2.38	2.88	1.00
time (sec)	N/A	0.016	0.009	0.059	0.262	0.413	0.087	0.422	2.112

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	36	37	48	0	38	26
N.S.	1	1.00	2.32	1.64	1.68	2.18	0.00	1.73	1.18
time (sec)	N/A	0.026	0.008	0.086	0.271	0.383	0.000	0.408	0.077

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	17	29	0	17	13
N.S.	1	1.00	1.11	0.95	0.89	1.53	0.00	0.89	0.68
time (sec)	N/A	0.030	0.006	0.075	0.269	0.385	0.000	0.390	2.029

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	75	52	51	72	0	47	39
N.S.	1	1.00	2.14	1.49	1.46	2.06	0.00	1.34	1.11
time (sec)	N/A	0.035	0.009	0.102	0.266	0.386	0.000	0.411	0.082

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	143	94	87	225	0	99	100
N.S.	1	1.00	1.83	1.21	1.12	2.88	0.00	1.27	1.28
time (sec)	N/A	0.058	0.259	0.234	0.468	0.414	0.000	0.418	2.132

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	116	57	53	152	0	59	65
N.S.	1	1.00	2.15	1.06	0.98	2.81	0.00	1.09	1.20
time (sec)	N/A	0.049	0.191	0.165	0.491	0.432	0.000	0.412	0.097

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	34	30	95	0	30	28
N.S.	1	1.00	2.50	0.94	0.83	2.64	0.00	0.83	0.78
time (sec)	N/A	0.035	0.179	0.129	0.474	0.398	0.000	0.415	0.085

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	18	17	73	66	17	18
N.S.	1	1.00	1.00	0.69	0.65	2.81	2.54	0.65	0.69
time (sec)	N/A	0.018	0.025	0.065	0.487	0.409	0.411	0.410	2.227

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	50	56	48	113	0	50	853
N.S.	1	1.00	1.19	1.33	1.14	2.69	0.00	1.19	20.31
time (sec)	N/A	0.033	0.050	0.133	0.477	0.445	0.000	0.420	2.666

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	140	95	105	274	0	103	1138
N.S.	1	1.00	2.26	1.53	1.69	4.42	0.00	1.66	18.35
time (sec)	N/A	0.059	0.569	0.207	0.478	0.456	0.000	0.432	2.589

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	204	155	200	592	0	178	833
N.S.	1	1.00	2.17	1.65	2.13	6.30	0.00	1.89	8.86
time (sec)	N/A	0.093	1.505	0.248	0.467	0.494	0.000	0.404	5.277

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	77	94	112	285	0	119	681
N.S.	1	1.00	0.88	1.07	1.27	3.24	0.00	1.35	7.74
time (sec)	N/A	0.127	0.207	0.181	0.475	0.457	0.000	0.421	2.683

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	61	62	211	0	80	126
N.S.	1	1.00	0.87	1.02	1.03	3.52	0.00	1.33	2.10
time (sec)	N/A	0.071	0.111	0.140	0.475	0.455	0.000	0.423	2.453

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	36	33	177	0	50	108
N.S.	1	1.00	0.92	0.90	0.82	4.42	0.00	1.25	2.70
time (sec)	N/A	0.038	0.079	0.086	0.482	0.450	0.000	0.399	2.370

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	10924	37	24
N.S.	1	1.00	0.97	0.70	0.67	5.43	364.13	1.23	0.80
time (sec)	N/A	0.013	0.066	0.066	0.469	0.437	19.413	0.416	2.384

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	39	38	228	0	55	34
N.S.	1	1.00	0.98	0.95	0.93	5.56	0.00	1.34	0.83
time (sec)	N/A	0.037	0.107	0.148	0.476	0.455	0.000	0.436	2.302

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	57	70	396	0	90	67
N.S.	1	1.00	0.97	0.93	1.15	6.49	0.00	1.48	1.10
time (sec)	N/A	0.055	0.241	0.180	0.480	0.464	0.000	0.430	2.335

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	83	127	610	0	156	101
N.S.	1	1.00	1.01	0.93	1.43	6.85	0.00	1.75	1.13
time (sec)	N/A	0.073	0.426	0.201	0.475	0.447	0.000	0.428	2.367

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	79	80	89	71	85	60	75
N.S.	1	1.00	0.81	0.82	0.91	0.72	0.87	0.61	0.77
time (sec)	N/A	0.064	0.103	0.133	0.472	0.414	0.543	0.393	0.307

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	6	8	14	6	4
N.S.	1	1.00	1.00	1.75	1.50	2.00	3.50	1.50	1.00
time (sec)	N/A	0.010	0.006	0.040	0.260	0.441	0.204	0.410	2.244

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	14	14	25	34	14	10
N.S.	1	1.00	1.31	1.08	1.08	1.92	2.62	1.08	0.77
time (sec)	N/A	0.010	0.006	0.051	0.270	0.406	0.477	0.412	2.249

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	20	20	37	54	20	17
N.S.	1	1.00	1.29	0.95	0.95	1.76	2.57	0.95	0.81
time (sec)	N/A	0.012	0.006	0.068	0.265	0.412	1.278	0.407	2.237

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	111	78	91	259	0	96	86
N.S.	1	1.00	1.42	1.00	1.17	3.32	0.00	1.23	1.10
time (sec)	N/A	0.056	0.453	0.235	0.487	0.421	0.000	0.414	0.134

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	86	50	67	191	0	65	51
N.S.	1	1.00	1.54	0.89	1.20	3.41	0.00	1.16	0.91
time (sec)	N/A	0.047	0.213	0.164	0.483	0.462	0.000	0.417	2.312

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	50	134	0	41	30
N.S.	1	1.00	1.00	0.87	1.32	3.53	0.00	1.08	0.79
time (sec)	N/A	0.037	0.037	0.127	0.474	0.439	0.000	0.395	0.101

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	21	39	95	55508	31	21
N.S.	1	1.00	1.00	0.72	1.34	3.28	1914.07	1.07	0.72
time (sec)	N/A	0.020	0.015	0.070	0.481	0.427	69.002	0.403	0.091

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	93	47	64	119	0	57	414
N.S.	1	1.00	2.27	1.15	1.56	2.90	0.00	1.39	10.10
time (sec)	N/A	0.035	0.159	0.127	0.476	0.421	0.000	0.414	2.500

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	152	82	92	186	0	85	483
N.S.	1	1.00	2.58	1.39	1.56	3.15	0.00	1.44	8.19
time (sec)	N/A	0.065	0.429	0.195	0.487	0.477	0.000	0.407	2.527

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	215	137	145	270	0	127	969
N.S.	1	1.00	2.39	1.52	1.61	3.00	0.00	1.41	10.77
time (sec)	N/A	0.109	1.335	0.253	0.476	0.449	0.000	0.412	2.639

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	76	92	97	273	0	104	1036
N.S.	1	1.00	0.87	1.06	1.11	3.14	0.00	1.20	11.91
time (sec)	N/A	0.129	0.239	0.194	0.476	0.441	0.000	0.410	2.694

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	59	54	213	0	72	291
N.S.	1	1.00	0.87	0.98	0.90	3.55	0.00	1.20	4.85
time (sec)	N/A	0.065	0.138	0.131	0.473	0.457	0.000	0.411	2.612

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	36	34	31	183	0	48	425
N.S.	1	1.00	0.95	0.89	0.82	4.82	0.00	1.26	11.18
time (sec)	N/A	0.043	0.090	0.079	0.476	0.423	0.000	0.409	2.549

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	21	20	163	10924	37	24
N.S.	1	1.00	0.97	0.70	0.67	5.43	364.13	1.23	0.80
time (sec)	N/A	0.013	0.054	0.000	0.480	0.412	19.213	0.407	0.002

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	33	32	216	0	36	30
N.S.	1	1.00	1.03	0.89	0.86	5.84	0.00	0.97	0.81
time (sec)	N/A	0.040	0.090	0.128	0.486	0.415	0.000	0.422	2.379

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	49	48	276	0	71	51
N.S.	1	1.00	0.98	0.88	0.86	4.93	0.00	1.27	0.91
time (sec)	N/A	0.060	0.166	0.179	0.474	0.457	0.000	0.406	2.309

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	78	74	348	0	104	84
N.S.	1	1.00	1.01	0.99	0.94	4.41	0.00	1.32	1.06
time (sec)	N/A	0.071	0.375	0.240	0.474	0.481	0.000	0.411	2.304

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	70	60	72	326	0	69	52
N.S.	1	1.00	1.08	0.92	1.11	5.02	0.00	1.06	0.80
time (sec)	N/A	0.035	0.261	0.126	0.473	0.453	0.000	0.401	2.341

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	106	117	186	616	0	149	123
N.S.	1	1.00	0.99	1.09	1.74	5.76	0.00	1.39	1.15
time (sec)	N/A	0.084	0.738	0.224	0.470	0.466	0.000	0.400	2.440

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	15	14	13	31	63	46	26
N.S.	1	1.00	0.44	0.41	0.38	0.91	1.85	1.35	0.76
time (sec)	N/A	0.009	0.030	0.051	0.475	0.430	0.235	0.390	2.325

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	27	26	57	218	59	40
N.S.	1	1.00	0.64	0.49	0.47	1.04	3.96	1.07	0.73
time (sec)	N/A	0.018	0.084	0.059	0.477	0.400	1.091	0.403	2.170

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	51	35	41	81	439	68	53
N.S.	1	1.00	0.72	0.49	0.58	1.14	6.18	0.96	0.75
time (sec)	N/A	0.035	0.138	0.066	0.474	0.435	3.582	0.399	2.149

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	10	4	0	24	10
N.S.	1	1.00	1.00	1.08	0.83	0.33	0.00	2.00	0.83
time (sec)	N/A	0.012	0.009	0.237	0.480	0.397	0.000	0.400	0.035

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	12	1	0	28	39
N.S.	1	1.00	1.00	1.00	0.86	0.07	0.00	2.00	2.79
time (sec)	N/A	0.010	0.007	0.304	0.491	0.378	0.000	0.404	2.295

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	19	11	11	0	45	-1
N.S.	1	1.00	0.79	0.66	0.38	0.38	0.00	1.55	-0.03
time (sec)	N/A	0.014	0.030	0.242	0.517	0.389	0.000	0.410	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	0	1	0	55	-1
N.S.	1	1.00	0.76	0.64	0.00	0.03	0.00	1.67	-0.03
time (sec)	N/A	0.015	0.032	0.279	0.000	0.392	0.000	0.424	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	28	14	35	19	0	21	-1
N.S.	1	1.00	1.87	0.93	2.33	1.27	0.00	1.40	-0.07
time (sec)	N/A	0.012	0.020	0.244	0.511	0.403	0.000	0.412	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	34	17	0	0	27	-1
N.S.	1	1.00	1.76	2.00	1.00	0.00	0.00	1.59	-0.06
time (sec)	N/A	0.012	0.012	0.213	0.531	0.000	0.000	0.411	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	51	37	300	44	0	78	-1
N.S.	1	1.00	1.59	1.16	9.38	1.38	0.00	2.44	-0.03
time (sec)	N/A	0.014	0.057	0.349	0.518	0.383	0.000	0.421	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	53	52	284	0	0	90	-1
N.S.	1	1.00	1.47	1.44	7.89	0.00	0.00	2.50	-0.03
time (sec)	N/A	0.016	0.036	0.306	0.513	0.000	0.000	0.438	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	10	0	0	7
N.S.	1	1.00	1.22	4.56	0.00	1.11	0.00	0.00	0.78
time (sec)	N/A	0.006	0.025	0.319	0.000	0.082	0.000	0.000	0.009

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	75	0	112	0	0	-1
N.S.	1	1.00	1.06	2.34	0.00	3.50	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.045	0.391	0.000	0.078	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	49	0	12	0	0	-1
N.S.	1	1.00	1.10	1.17	0.00	0.29	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.087	0.255	0.000	0.096	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	101	0	10	0	0	-1
N.S.	1	1.00	0.91	2.35	0.00	0.23	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.058	0.490	0.000	0.087	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	110	0	145	0	0	-1
N.S.	1	1.00	0.74	1.24	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.081	0.471	0.000	0.092	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	123	192	0	12	0	0	-1
N.S.	1	1.00	1.02	1.59	0.00	0.10	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.525	0.414	0.000	0.127	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	41	0	87	0	0	-1
N.S.	1	1.00	1.22	4.56	0.00	9.67	0.00	0.00	-0.11
time (sec)	N/A	0.006	0.037	0.317	0.000	0.108	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	62	0	41	0	0	-1
N.S.	1	1.00	1.03	1.94	0.00	1.28	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.045	0.279	0.000	0.083	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	48	0	276	0	0	-1
N.S.	1	1.00	1.10	1.14	0.00	6.57	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.069	0.274	0.000	0.102	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	35	70	0	247	0	0	-1
N.S.	1	1.00	1.09	2.19	0.00	7.72	0.00	0.00	-0.03
time (sec)	N/A	0.013	0.069	0.433	0.000	0.126	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	101	0	168	0	0	-1
N.S.	1	1.00	0.77	1.80	0.00	3.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.048	0.530	0.000	0.114	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	75	73	0	775	0	0	-1
N.S.	1	1.00	0.96	0.94	0.00	9.94	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.193	0.399	0.000	0.160	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	33	8	49	0	23	-1
N.S.	1	1.00	1.00	3.67	0.89	5.44	0.00	2.56	-0.11
time (sec)	N/A	0.015	0.009	0.274	0.470	0.383	0.000	0.433	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	57	11	89	0	0	-1
N.S.	1	1.00	1.00	3.80	0.73	5.93	0.00	0.00	-0.07
time (sec)	N/A	0.018	0.028	0.321	0.470	0.393	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	53	8	39	0	29	-1
N.S.	1	1.00	1.00	5.89	0.89	4.33	0.00	3.22	-0.11
time (sec)	N/A	0.017	0.012	0.385	0.473	0.385	0.000	0.436	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	121	1233	0	809	0	307	926
N.S.	1	1.00	0.25	2.53	0.00	1.66	0.00	0.63	1.90
time (sec)	N/A	0.756	0.256	0.207	0.000	0.529	0.000	0.425	2.655

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	109	72	0	817	0	299	938
N.S.	1	1.00	1.08	0.71	0.00	8.09	0.00	2.96	9.29
time (sec)	N/A	0.080	0.212	0.137	0.000	0.499	0.000	0.444	2.596

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	45	167	0	3830	0	170	214
N.S.	1	1.00	0.15	0.57	0.00	13.12	0.00	0.58	0.73
time (sec)	N/A	0.132	0.080	0.211	0.000	18.437	0.000	0.539	2.730

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	24	21	20	43	78	53	20
N.S.	1	1.00	0.53	0.47	0.44	0.96	1.73	1.18	0.44
time (sec)	N/A	0.014	0.062	0.065	0.484	0.407	0.629	0.397	2.164

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	130	150	0	0	0	0	1520
N.S.	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	3.08
time (sec)	N/A	0.649	0.225	0.515	0.000	0.000	0.000	0.000	8.818

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	60	0	15483	0	0	184
N.S.	1	1.00	0.85	0.35	0.00	90.54	0.00	0.00	1.08
time (sec)	N/A	0.177	0.243	0.427	0.000	1.961	0.000	0.000	3.081

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	172	76	0	665467	0	0	216
N.S.	1	1.00	0.70	0.31	0.00	2716.19	0.00	0.00	0.88
time (sec)	N/A	0.334	0.286	0.465	0.000	6.450	0.000	0.000	3.421

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	130	148	0	0	0	0	1518
N.S.	1	1.00	0.26	0.30	0.00	0.00	0.00	0.00	3.07
time (sec)	N/A	0.471	0.192	0.493	0.000	0.000	0.000	0.000	7.832

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	146	62	0	16679	0	0	184
N.S.	1	1.00	0.83	0.35	0.00	95.31	0.00	0.00	1.05
time (sec)	N/A	0.176	0.187	0.400	0.000	1.966	0.000	0.000	3.120

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	78	0	643291	0	0	216
N.S.	1	1.00	0.81	0.37	0.00	3020.15	0.00	0.00	1.01
time (sec)	N/A	0.147	0.231	0.461	0.000	6.609	0.000	0.000	3.416

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	378	62	0	0	0	0	535
N.S.	1	1.00	1.70	0.28	0.00	0.00	0.00	0.00	2.40
time (sec)	N/A	0.388	0.135	0.158	0.000	0.000	0.000	0.000	2.778

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	103	79	73	72	138	0	185	99
N.S.	1	1.24	0.95	0.88	0.87	1.66	0.00	2.23	1.19
time (sec)	N/A	0.076	0.177	0.117	0.477	0.477	0.000	0.458	2.389

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	141	67	0	0	0	0	1025
N.S.	1	1.00	1.09	0.52	0.00	0.00	0.00	0.00	7.95
time (sec)	N/A	0.125	0.164	0.354	0.000	0.000	0.000	0.000	3.108

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	378	62	0	0	0	0	403
N.S.	1	1.00	1.84	0.30	0.00	0.00	0.00	0.00	1.97
time (sec)	N/A	0.329	0.129	0.137	0.000	0.000	0.000	0.000	2.447

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	117	171	0	0	728	199	95
N.S.	1	1.00	1.65	2.41	0.00	0.00	10.25	2.80	1.34
time (sec)	N/A	0.083	0.315	0.174	0.000	0.000	9.010	0.489	2.293

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	64	186	0	3963	0	222	241
N.S.	1	1.00	0.72	2.09	0.00	44.53	0.00	2.49	2.71
time (sec)	N/A	0.051	0.173	0.213	0.000	18.294	0.000	0.650	2.267

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	19	19	0	16	9
N.S.	1	1.00	1.00	0.94	1.12	1.12	0.00	0.94	0.53
time (sec)	N/A	0.020	0.011	0.081	0.264	0.541	0.000	0.483	2.163

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	43	95	90	0	38	-1
N.S.	1	1.00	1.00	1.08	2.38	2.25	0.00	0.95	-0.02
time (sec)	N/A	0.040	0.031	0.065	0.485	0.526	0.000	0.463	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	47	17	47	21	0	45	-1
N.S.	1	1.00	2.35	0.85	2.35	1.05	0.00	2.25	-0.05
time (sec)	N/A	0.032	0.031	0.335	0.478	0.405	0.000	0.458	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	81	67	0	24	-1
N.S.	1	1.00	1.00	1.20	3.24	2.68	0.00	0.96	-0.04
time (sec)	N/A	0.037	0.016	0.059	0.477	0.456	0.000	0.493	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	60	16	0	27	-1
N.S.	1	1.00	1.00	0.91	5.45	1.45	0.00	2.45	-0.09
time (sec)	N/A	0.025	0.012	0.066	0.477	0.408	0.000	0.441	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	44	8	39	17	0	33	-1
N.S.	1	1.00	4.89	0.89	4.33	1.89	0.00	3.67	-0.11
time (sec)	N/A	0.040	0.023	0.224	0.478	0.383	0.000	0.446	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	217	132	151	1690	0	143	1281
N.S.	1	1.00	1.42	0.86	0.99	11.05	0.00	0.93	8.37
time (sec)	N/A	0.131	0.329	0.195	0.484	18.860	0.000	0.480	5.211

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	52	123	0	38	-1
N.S.	1	1.00	1.00	0.76	1.16	2.73	0.00	0.84	-0.02
time (sec)	N/A	0.052	0.028	0.815	0.480	1.764	0.000	0.439	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	39	0	0	24	-1
N.S.	1	1.00	1.00	0.75	1.39	0.00	0.00	0.86	-0.04
time (sec)	N/A	0.044	0.016	0.072	0.494	0.000	0.000	0.484	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	44	43	90	0	38	-1
N.S.	1	1.00	1.00	0.98	0.96	2.00	0.00	0.84	-0.02
time (sec)	N/A	0.051	0.039	0.070	0.272	0.482	0.000	0.475	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	31	0	67	0	24	-1
N.S.	1	1.00	1.00	1.11	0.00	2.39	0.00	0.86	-0.04
time (sec)	N/A	0.045	0.017	0.063	0.000	0.445	0.000	0.417	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	39	58	97	0	46	-1
N.S.	1	1.00	0.98	0.83	1.23	2.06	0.00	0.98	-0.02
time (sec)	N/A	0.050	0.035	0.570	0.489	0.385	0.000	0.490	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	42	74	0	27	-1
N.S.	1	1.00	1.00	0.83	1.45	2.55	0.00	0.93	-0.03
time (sec)	N/A	0.051	0.017	0.046	0.474	0.438	0.000	0.505	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [68] had the largest ratio of [21]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	16	0.188
2	A	3	2	1.00	16	0.125
3	A	3	3	1.00	16	0.188
4	A	2	2	1.00	16	0.125
5	A	2	2	1.00	16	0.125
6	A	2	2	1.00	14	0.143
7	A	3	3	1.00	14	0.214
8	A	3	2	1.00	16	0.125
9	A	4	3	1.00	16	0.188
10	A	4	3	1.00	15	0.200
11	A	4	3	1.00	15	0.200
12	A	3	3	1.00	15	0.200
13	A	2	2	1.00	13	0.154
14	A	4	4	1.00	13	0.308
15	A	5	5	1.00	15	0.333
16	A	6	6	1.00	15	0.400
17	A	6	6	1.00	15	0.400
18	A	5	5	1.00	15	0.333
19	A	4	4	1.00	15	0.267
20	A	2	2	1.00	10	0.200
21	A	3	3	1.00	15	0.200
22	A	4	3	1.00	15	0.200
23	A	4	3	1.00	15	0.200
24	A	7	7	1.00	13	0.538
25	A	3	3	1.00	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	10	0.200
27	A	3	2	1.00	10	0.200
28	A	4	3	1.00	15	0.200
29	A	4	3	1.00	15	0.200
30	A	3	3	1.00	15	0.200
31	A	2	2	1.00	13	0.154
32	A	4	4	1.00	13	0.308
33	A	5	5	1.00	15	0.333
34	A	6	6	1.00	15	0.400
35	A	6	6	1.00	15	0.400
36	A	5	5	1.00	15	0.333
37	A	3	3	1.00	15	0.200
38	A	2	2	1.00	10	0.200
39	A	3	3	1.00	15	0.200
40	A	4	3	1.00	15	0.200
41	A	4	3	1.00	15	0.200
42	A	4	4	1.00	10	0.400
43	A	5	5	1.00	10	0.500
44	A	2	2	1.00	8	0.250
45	A	4	4	1.00	8	0.500
46	A	5	5	1.00	8	0.625
47	A	3	3	1.00	12	0.250
48	A	3	3	1.00	10	0.300
49	A	4	4	1.00	12	0.333
50	A	4	4	1.00	10	0.400
51	A	3	3	1.00	12	0.250
52	A	3	3	1.00	10	0.300
53	A	4	4	1.00	12	0.333
54	A	4	4	1.00	10	0.400
55	A	1	1	1.00	10	0.100
56	A	2	2	1.00	12	0.167
57	A	2	2	1.00	12	0.167
58	A	4	4	1.00	10	0.400
59	A	6	6	1.00	12	0.500
60	A	6	6	1.00	12	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	1	1	1.00	10	0.100
62	A	2	2	1.00	12	0.167
63	A	2	2	1.00	12	0.167
64	A	3	3	1.00	10	0.300
65	A	4	4	1.00	12	0.333
66	A	4	4	1.00	12	0.333
67	A	2	2	1.00	13	0.154
68	A	2	2	1.00	21	0.095
69	A	2	2	1.00	15	0.133
70	A	10	6	1.00	10	0.600
71	A	4	3	1.00	11	0.273
72	A	10	6	1.00	8	0.750
73	A	3	3	1.00	10	0.300
74	A	12	3	1.00	10	0.300
75	A	7	3	1.00	10	0.300
76	A	9	3	1.00	10	0.300
77	A	12	3	1.00	11	0.273
78	A	7	3	1.00	11	0.273
79	A	9	3	1.00	11	0.273
80	A	11	5	1.00	8	0.625
81	A	7	3	1.24	8	0.375
82	A	9	3	1.00	8	0.375
83	A	11	5	1.00	10	0.500
84	A	8	6	1.00	10	0.600
85	A	10	6	1.00	10	0.600
86	A	4	4	1.00	11	0.364
87	A	4	4	1.00	15	0.267
88	A	5	5	1.00	15	0.333
89	A	3	3	1.00	15	0.200
90	A	3	3	1.00	13	0.231
91	A	4	4	1.00	15	0.267
92	A	11	10	1.00	15	0.667
93	A	5	5	1.00	15	0.333
94	A	4	4	1.00	15	0.267
95	A	5	5	1.00	15	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	15	0.267
97	A	5	5	1.00	15	0.333
98	A	4	4	1.00	15	0.267

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{\sin^6(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a}-\frac{3\cos(x)\sin(x)}{8a}-\frac{\cos(x)\sin^3(x)}{4a}$$

```
[Out] 3/8*x/a-3/8*cos(x)*sin(x)/a-1/4*cos(x)*sin(x)^3/a
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3254, 2715, 8}

$$\frac{3x}{8a}-\frac{\sin^3(x)\cos(x)}{4a}-\frac{3\sin(x)\cos(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a - a*Cos[x]^2),x]
```

```
[Out] (3*x)/(8*a) - (3*Cos[x]*Sin[x])/(8*a) - (Cos[x]*Sin[x]^3)/(4*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(x)}{a-a\cos^2(x)}\,dx = \frac{\int \sin^4(x)\,dx}{a}$$
$$= -\frac{\cos(x)\sin^3(x)}{4a} + \frac{3\int \sin^2(x)\,dx}{4a}$$
$$= -\frac{3\cos(x)\sin(x)}{8a} - \frac{\cos(x)\sin^3(x)}{4a} + \frac{3\int 1\,dx}{8a}$$
$$= \frac{3x}{8a} - \frac{3\cos(x)\sin(x)}{8a} - \frac{\cos(x)\sin^3(x)}{4a}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a - a*Cos[x]^2),x]
```

```
[Out] ((3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32)/a
```

Maple [A]
time = 0.09, size = 31, normalized size = 0.94

method	result
risch	$\frac{3x}{8a} + \frac{\sin(4x)}{32a} - \frac{\sin(2x)}{4a}$
default	$\frac{\frac{-\frac{5\left(\tan^3(x)\right)}{8} - \frac{3\tan(x)}{8}}{\left(1+\tan^2(x)\right)^2} + \frac{3\arctan(\tan(x))}{8}}{a}$
norman	$\frac{-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{4a} - \frac{17\left(\tan^4\left(\frac{x}{2}\right)\right)}{4a} - \frac{7\left(\tan^6\left(\frac{x}{2}\right)\right)}{2a} + \frac{7\left(\tan^8\left(\frac{x}{2}\right)\right)}{2a} + \frac{17\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{4a} + \frac{3\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{4a} + \frac{3x\tan\left(\frac{x}{2}\right)}{8a} + \frac{9x\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a} + \frac{45x\left(\tan^5\left(\frac{x}{2}\right)\right)}{8a} +}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6\tan\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*((-5/8*tan(x)^3-3/8*tan(x))/(tan(x)^2+1)^2+3/8*arctan(tan(x)))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.12

$$-\frac{5\,\tan\left(x\right)^3 + 3\,\tan\left(x\right)}{8\,\left(a\tan\left(x\right)^4 + 2\,a\tan\left(x\right)^2 + a\right)} + \frac{3\,x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(5*tan(x)^3 + 3*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

Fricas [A]
time = 0.38, size = 23, normalized size = 0.70

$$\frac{\left(2\,\cos\left(x\right)^{3}-5\,\cos\left(x\right)\right)\sin\left(x\right)+3\,x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*cos(x)^3 - 5*cos(x))*sin(x) + 3*x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(29) = 58.
time = 2.26, size = 473, normalized size = 14.33

$$\frac{3x\tan^{8}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^{6}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{18x\tan^{4}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^{2}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{3x}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{6\tan^{7}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}+\frac{22\tan^{5}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}-\frac{22\tan^{3}\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}-\frac{6\tan\left(\frac{x}{2}\right)}{8a\tan^{8}\left(\frac{x}{2}\right)+32a\tan^{6}\left(\frac{x}{2}\right)+48a\tan^{4}\left(\frac{x}{2}\right)+32a\tan^{2}\left(\frac{x}{2}\right)+8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a-a*cos(x)**2),x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**
6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*tan(
x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 1
2*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(
x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/2)**7/(8*a*tan(x/2)**8 + 32*a*t
an(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 22*tan(x/2)**5/(8
*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8
*a) - 22*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
 + 32*a*tan(x/2)**2 + 8*a) - 6*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6
 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

Giac [A]
time = 0.41, size = 31, normalized size = 0.94

$$\frac{3\,x}{8\,a}-\frac{5\,\tan\left(x\right)^{3}+3\,\tan\left(x\right)}{8\,{\left(\tan\left(x\right)^{2}+1\right)}^{2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] 3/8*x/a - 1/8*(5*tan(x)^3 + 3*tan(x))/((tan(x)^2 + 1)^2*a)
```

Mupad [B]
time = 2.11, size = 25, normalized size = 0.76

$$\frac{\sin(4x)}{32a} - \frac{\sin(2x)}{4a} + \frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a - a*cos(x)^2),x)
```

```
[Out] sin(4*x)/(32*a) - sin(2*x)/(4*a) + (3*x)/(8*a)
```

3.2 $\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\cos(x)}{a}+\frac{\cos^3(x)}{3a}$$

```
[Out] -cos(x)/a+1/3*cos(x)^3/a
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2713}

$$\frac{\cos^3(x)}{3a}-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a - a*Cos[x]^2),x]
```

```
[Out] -(Cos[x]/a) + Cos[x]^3/(3*a)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \sin^3(x)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)}{a}\\
&= -\frac{\cos(x)}{a}+\frac{\cos^3(x)}{3a}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{-\frac{3\cos(x)}{4}+\frac{1}{12}\cos(3x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a - a*Cos[x]^2),x]
```

```
[Out] ((-3*Cos[x])/4 + Cos[3*x]/12)/a
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.84

method	result	size
default	$\frac{\frac{\left(\cos^{3}(x)\right)}{3}-\cos(x)}{a}$	16
risch	$-\frac{3\cos(x)}{4a}+\frac{\cos(3x)}{12a}$	18
norman	$\frac{-\frac{4\tan\left(\frac{x}{2}\right)}{3a}-\frac{20\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3a}-\frac{4\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{a}-\frac{28\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{3a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{5}\tan\left(\frac{x}{2}\right)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/3*cos(x)^3-cos(x))
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.74

$$\frac{\cos(x)^{3}-3\cos(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(cos(x)^3 - 3*cos(x))/a
```

Fricas [A]
time = 0.38, size = 14, normalized size = 0.74

$$\frac{\cos(x)^{3}-3\cos(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^3 - 3*cos(x))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(12) = 24$.
time = 1.37, size = 78, normalized size = 4.11

$$-\frac{12\tan^{2}\left(\frac{x}{2}\right)}{3a\tan^{6}\left(\frac{x}{2}\right)+9a\tan^{4}\left(\frac{x}{2}\right)+9a\tan^{2}\left(\frac{x}{2}\right)+3a}-\frac{4}{3a\tan^{6}\left(\frac{x}{2}\right)+9a\tan^{4}\left(\frac{x}{2}\right)+9a\tan^{2}\left(\frac{x}{2}\right)+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a-a*cos(x)**2),x)
```

```
[Out] -12*tan(x/2)**2/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
 - 4/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
```

Giac [A]
time = 0.42, size = 14, normalized size = 0.74

$$\frac{\cos\left(x\right)^{3}-3\,\cos\left(x\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*(cos(x)^3 - 3*cos(x))/a
```

Mupad [B]
time = 2.04, size = 16, normalized size = 0.84

$$-\frac{3\,\cos\left(x\right)-\cos\left(x\right)^{3}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a - a*cos(x)^2),x)
```

```
[Out] -(3*cos(x) - cos(x)^3)/(3*a)
```

3.3 $\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}$$

```
[Out] 1/2*x/a-1/2*cos(x)*sin(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3254, 2715, 8}

$$\frac{x}{2a}-\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a - a*Cos[x]^2),x]
```

```
[Out] x/(2*a) - (Cos[x]*Sin[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \sin^2(x)\,dx}{a}\\ &= -\frac{\cos(x)\sin(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}-\frac{\cos(x)\sin(x)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}-\frac{1}{4}\sin(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a - a*Cos[x]^2),x]
[Out] (x/2 - Sin[2*x]/4)/a
```

Maple [A]
time = 0.06, size = 23, normalized size = 1.15

method	result	size
risch	$\frac{x}{2a}-\frac{\sin(2x)}{4a}$	17
default	$\frac{-\frac{\tan(x)}{2(1+\tan^2(x))}+\frac{\arctan(\tan(x))}{2}}{a}$	23
norman	$\frac{\frac{\tan^6(\frac{x}{2})}{a}+\frac{\tan^8(\frac{x}{2})}{a}-\frac{\tan^2(\frac{x}{2})}{a}-\frac{\tan^4(\frac{x}{2})}{a}+\frac{x\tan(\frac{x}{2})}{2a}+\frac{2x(\tan^3(\frac{x}{2}))}{a}+\frac{3x(\tan^5(\frac{x}{2}))}{a}+\frac{2x(\tan^7(\frac{x}{2}))}{a}+\frac{x(\tan^9(\frac{x}{2}))}{2a}}{(1+\tan^2(\frac{x}{2}))^4\tan(\frac{x}{2})}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(-1/2*tan(x)/(tan(x)^2+1)+1/2*arctan(tan(x)))
```

Maxima [A]
time = 0.48, size = 21, normalized size = 1.05

$$\frac{x}{2\,a}-\frac{\tan\left(x\right)}{2\,\left(a\tan\left(x\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] 1/2*x/a - 1/2*tan(x)/(a*tan(x)^2 + a)
```

Fricas [A]
time = 0.38, size = 14, normalized size = 0.70

$$-\frac{\cos\left(x\right)\sin\left(x\right)-x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*sin(x) - x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(14) = 28$.
time = 0.81, size = 153, normalized size = 7.65

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{x}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{2\tan^3\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}-\frac{2\tan\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a-a*cos(x)**2),x)
```

```
[Out] x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/(
2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x/2
)**2 + 2*a) + 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) - 2*t
an(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)
```

Giac [A]
time = 0.43, size = 22, normalized size = 1.10

$$\frac{x}{2\,a}-\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^2+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*x/a - 1/2*tan(x)/((tan(x)^2 + 1)*a)
```

Mupad [B]
time = 2.04, size = 15, normalized size = 0.75

$$\frac{2\,x-\sin\left(2\,x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a - a*cos(x)^2),x)
```

```
[Out] (2*x - sin(2*x))/(4*a)
```

3.4 $\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=7

$$-\frac{\cos(x)}{a}$$

[Out] -cos(x)/a

Rubi [A]
time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 2718}

$$-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^3/(a - a*Cos[x]^2),x]

[Out] -(Cos[x]/a)

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \sin(x)\,dx}{a}\\ &= -\frac{\cos(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a - a*Cos[x]^2),x]
[Out] -(Cos[x]/a)
```

Maple [A]
time = 0.05, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$-\frac{\cos(x)}{a}$	8
default	$-\frac{\cos(x)}{a}$	8
risch	$-\frac{\cos(x)}{a}$	8
norman	$\frac{\frac{2\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{a}+\frac{4\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}\tan\left(\frac{x}{2}\right)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -cos(x)/a
```

Maxima [A]
time = 0.26, size = 7, normalized size = 1.00

$$-\frac{\cos\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] -cos(x)/a
```

Fricas [A]
time = 0.38, size = 7, normalized size = 1.00

$$-\frac{\cos\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] -cos(x)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.
time = 0.46, size = 12, normalized size = 1.71

$$-\frac{2}{a\tan^{2}\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a-a*cos(x)**2),x)
```

```
[Out] -2/(a*tan(x/2)**2 + a)
```

Giac [A]
time = 0.41, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -cos(x)/a
```

Mupad [B]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{\cos(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a - a*cos(x)^2),x)
```

```
[Out] -cos(x)/a
```

3.5 $\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

```
[Out] x/a
```

Rubi [A]
time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3254, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a - a*Cos[x]^2),x]
```

```
[Out] x/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{a-a\cos^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a - a*Cos[x]^2),x]
[Out] x/a
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.06, size = 8, normalized size = 1.60

method	result	size
risch	$\frac{x}{a}$	6
default	$\frac{\arctan(\tan(x))}{a}$	8
norman	$\frac{\frac{x\tan\left(\frac{x}{2}\right)}{a}+\frac{x\left(\tan^5\left(\frac{x}{2}\right)\right)}{a}+\frac{2x\left(\tan^3\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*arctan(tan(x))
```

Maxima [A]
time = 0.48, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] x/a
```

Fricas [A]
time = 0.38, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] x/a
```

Sympy [A]
time = 0.26, size = 2, normalized size = 0.40

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a-a*cos(x)**2),x)
```

```
[Out] x/a
```

Giac [A]
time = 0.42, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] x/a
```

Mupad [B]
time = 2.05, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a - a*cos(x)^2),x)
```

```
[Out] x/a
```

3.6 $\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-\frac{\tanh^{-1}(\cos(x))}{a}$$

```
[Out] -arctanh(cos(x))/a
```

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3254, 3855}

$$-\frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a - a*Cos[x]^2),x]
```

```
[Out] -(ArcTanh[Cos[x]]/a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc(x)\,dx}{a}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 0.01, size = 21, normalized size = 2.62

$$\frac{-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a - a*Cos[x]^2),x]
[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]])/a
```

Maple [A]
time = 0.06, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}(\cos(x))}{a}$	9
default	$-\frac{\operatorname{arctanh}(\cos(x))}{a}$	9
norman	$\frac{\ln(\tan(\frac{x}{2}))}{a}$	10
risch	$\frac{\ln(e^{ix}-1)}{a}-\frac{\ln(e^{ix}+1)}{a}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -arctanh(cos(x))/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 0.26, size = 21, normalized size = 2.62

$$-\frac{\log\left(\cos\left(x\right)+1\right)}{2\,a}+\frac{\log\left(\cos\left(x\right)-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] -1/2*log(cos(x) + 1)/a + 1/2*log(cos(x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.
time = 0.41, size = 22, normalized size = 2.75

$$-\frac{\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] -1/2*(log(1/2*cos(x) + 1/2) - log(-1/2*cos(x) + 1/2))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.
time = 0.09, size = 19, normalized size = 2.38

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2a}-\frac{\log\left(\cos\left(x\right)+1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)**2),x)
```

```
[Out] log(cos(x) - 1)/(2*a) - log(cos(x) + 1)/(2*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.
time = 0.42, size = 23, normalized size = 2.88

$$-\frac{\log\left(\cos\left(x\right)+1\right)}{2\,a}+\frac{\log\left(-\cos\left(x\right)+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x) + 1)/a + 1/2*log(-cos(x) + 1)/a
```

Mupad [B]
time = 2.11, size = 8, normalized size = 1.00

$$-\frac{\mathrm{atanh}(\cos\left(x\right))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a - a*cos(x)^2),x)
```

```
[Out] -atanh(cos(x))/a
```

3.7 $\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}$$

[Out] -1/2*arctanh(cos(x))/a-1/2*cot(x)*csc(x)/a

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3254, 3853, 3855}

$$-\frac{\tanh^{-1}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(a - a*Cos[x]^2),x]

[Out] -1/2*ArcTanh[Cos[x]]/a - (Cot[x]*Csc[x])/(2*a)

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc^3(x)\,dx}{a}\\ &= -\frac{\cot(x)\csc(x)}{2a}+\frac{\int \csc(x)\,dx}{2a}\\ &= -\frac{\tanh^{-1}(\cos(x))}{2a}-\frac{\cot(x)\csc(x)}{2a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.
time = 0.01, size = 51, normalized size = 2.32

$$\frac{-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a - a*Cos[x]^2),x]
[Out] (-1/8*Csc[x/2]^2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8)/a
```

Maple [A]
time = 0.09, size = 36, normalized size = 1.64

method	result	size
default	$\frac{\frac{1}{4\cos(x)+4}-\frac{\ln(\cos(x)+1)}{4}+\frac{1}{-4+4\cos(x)}+\frac{\ln(-1+\cos(x))}{4}}{a}$	36
norman	$\frac{-\frac{1}{8a}+\frac{\tan^4\left(\frac{x}{2}\right)}{8a}}{\tan\left(\frac{x}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a}$	36
risch	$\frac{e^{3ix}+e^{ix}}{\left(e^{2ix}-1\right)^2 a}-\frac{\ln\left(e^{ix}+1\right)}{2a}+\frac{\ln\left(e^{ix}-1\right)}{2a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/4/(cos(x)+1)-1/4*ln(cos(x)+1)+1/4/(-1+cos(x))+1/4*ln(-1+cos(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 0.27, size = 37, normalized size = 1.68

$$\frac{\cos(x)}{2\left(a\cos(x)^2-a\right)}-\frac{\log(\cos(x)+1)}{4\,a}+\frac{\log(\cos(x)-1)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*cos(x)/(a*cos(x)^2 - a) - 1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.
time = 0.38, size = 48, normalized size = 2.18

$$-\frac{\left(\cos\left(x\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-\left(\cos\left(x\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-2\cos\left(x\right)}{4\left(a\cos\left(x\right)^{2}-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(a*cos(x)^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc\left(x\right)}{\cos^{2}\left(x\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)**2),x)
```

```
[Out] -Integral(csc(x)/(cos(x)**2 - 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.41, size = 38, normalized size = 1.73

$$-\frac{\log\left(\cos\left(x\right)+1\right)}{4\,a}+\frac{\log\left(-\cos\left(x\right)+1\right)}{4\,a}+\frac{\cos\left(x\right)}{2\left(\cos\left(x\right)^{2}-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a + 1/2*cos(x)/((cos(x)^2 - 1
)*a)
```

Mupad [B]
time = 0.08, size = 26, normalized size = 1.18

$$-\frac{\cos\left(x\right)}{2\left(a-a\cos\left(x\right)^{2}\right)}-\frac{\mathrm{atanh}\left(\cos\left(x\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a - a*cos(x)^2)),x)
```

```
[Out] - cos(x)/(2*(a - a*cos(x)^2)) - atanh(cos(x))/(2*a)
```

3.8 $\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\cot(x)}{a}-\frac{\cot^3(x)}{3a}$$

```
[Out] -cot(x)/a-1/3*cot(x)^3/a
```

Rubi [A]

time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3254, 3852}

$$-\frac{\cot^3(x)}{3a}-\frac{\cot(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a - a*Cos[x]^2),x]
```

```
[Out] -(Cot[x]/a) - Cot[x]^3/(3*a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc^4(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(x)\right)}{a}\\ &= -\frac{\cot(x)}{a}-\frac{\cot^3(x)}{3a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.11

$$\frac{-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a - a*Cos[x]^2),x]
[Out] ((-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3)/a
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.95

method	result	size
default	$\frac{-\frac{1}{3\tan(x)^3}-\frac{1}{\tan(x)}}{a}$	18
risch	$\frac{4i(3\,e^{2ix}-1)}{3(e^{2ix}-1)^3a}$	25
norman	$\frac{-\frac{1}{24a}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8a}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{24a}}{\tan\left(\frac{x}{2}\right)^3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(-1/3/tan(x)^3-1/tan(x))
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.89

$$-\frac{3\,\tan(x)^2+1}{3\,a\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="maxima")
[Out] -1/3*(3*tan(x)^2 + 1)/(a*tan(x)^3)
```

Fricas [A]
time = 0.38, size = 29, normalized size = 1.53

$$-\frac{2\,\cos(x)^3-3\,\cos(x)}{3\,\left(a\cos(x)^2-a\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="fricas")
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((a*cos(x)^2 - a)*sin(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^2(x)}{\cos^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a-a*cos(x)**2),x)
[Out] -Integral(csc(x)**2/(cos(x)**2 - 1), x)/a
```

Giac [**A**]
time = 0.39, size = 17, normalized size = 0.89

$$-\frac{3\,\tan(x)^2+1}{3\,a\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a-a*cos(x)^2),x, algorithm="giac")
[Out] -1/3*(3*tan(x)^2 + 1)/(a*tan(x)^3)
```

Mupad [**B**]
time = 2.03, size = 13, normalized size = 0.68

$$-\frac{\cot(x)\,\left(\cot(x)^2+3\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a - a*cos(x)^2)),x)
[Out] -(cot(x)*(cot(x)^2 + 3))/(3*a)
```

3.9 $\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx$

Optimal. Leaf size=35

$$-\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{3\cot(x)\csc(x)}{8a}-\frac{\cot(x)\csc^3(x)}{4a}$$

```
[Out] -3/8*arctanh(cos(x))/a-3/8*cot(x)*csc(x)/a-1/4*cot(x)*csc(x)^3/a
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3254, 3853, 3855}

$$-\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{\cot(x)\csc^3(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a - a*Cos[x]^2),x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/(8*a) - (3*Cot[x]*Csc[x])/(8*a) - (Cot[x]*Csc[x]^3)/(4
*a)
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a-a\cos^2(x)}\,dx &= \frac{\int \csc^5(x)\,dx}{a} \\ &= -\frac{\cot(x)\csc^3(x)}{4a}+\frac{3\int \csc^3(x)\,dx}{4a} \\ &= -\frac{3\cot(x)\csc(x)}{8a}-\frac{\cot(x)\csc^3(x)}{4a}+\frac{3\int \csc(x)\,dx}{8a} \\ &= -\frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{3\cot(x)\csc(x)}{8a}-\frac{\cot(x)\csc^3(x)}{4a}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.
time = 0.01, size = 75, normalized size = 2.14

$$\frac{-\frac{3}{32}\csc^2\left(\frac{x}{2}\right)-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{3}{32}\sec^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a - a*Cos[x]^2),x]
[Out] ((-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]
])/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64)/a
```

Maple [A]
time = 0.10, size = 52, normalized size = 1.49

method	result	size
default	$\frac{\frac{1}{16(\cos(x)+1)^2}+\frac{3}{16(\cos(x)+1)}-\frac{3\ln(\cos(x)+1)}{16}-\frac{1}{16(-1+\cos(x))^2}+\frac{3}{16(-1+\cos(x))}+\frac{3\ln(-1+\cos(x))}{16}}{a}$	52
norman	$\frac{-\frac{1}{64a}-\frac{\tan^2\left(\frac{x}{2}\right)}{8a}+\frac{\tan^6\left(\frac{x}{2}\right)}{8a}+\frac{\tan^8\left(\frac{x}{2}\right)}{64a}}{\tan\left(\frac{x}{2}\right)^4}+\frac{3\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8a}$	58
risch	$\frac{3\,e^{7ix}-11\,e^{5ix}-11\,e^{3ix}+3\,e^{ix}}{4(e^{2ix}-1)^4 a}+\frac{3\ln(e^{ix}-1)}{8a}-\frac{3\ln(e^{ix}+1)}{8a}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a-a*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a*(1/16/(cos(x)+1)^2+3/16/(cos(x)+1)-3/16*ln(cos(x)+1)-1/16/(-1+cos(x))^2
+3/16/(-1+cos(x))+3/16*ln(-1+cos(x)))
```

Maxima [A]
time = 0.27, size = 51, normalized size = 1.46

$$\frac{3\,\cos\left(x\right)^3-5\,\cos\left(x\right)}{8\,\left(a\cos\left(x\right)^4-2\,a\cos\left(x\right)^2+a\right)}-\frac{3\,\log\left(\cos\left(x\right)+1\right)}{16\,a}+\frac{3\,\log\left(\cos\left(x\right)-1\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(a*cos(x)^4 - 2*a*cos(x)^2 + a) - 3/16*log(cos(
x) + 1)/a + 3/16*log(cos(x) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.
time = 0.39, size = 72, normalized size = 2.06

$$\frac{6\cos(x)^3 - 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 3\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 10\cos(x)}{16\left(a\cos(x)^4 - 2a\cos(x)^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3*
(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(a*cos(x)^4
 - 2*a*cos(x)^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^3(x)}{\cos^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a-a*cos(x)**2),x)
```

```
[Out] -Integral(csc(x)**3/(cos(x)**2 - 1), x)/a
```

Giac [A]
time = 0.41, size = 47, normalized size = 1.34

$$-\frac{3\log(\cos(x)+1)}{16a} + \frac{3\log(-\cos(x)+1)}{16a} + \frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^2-1\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a-a*cos(x)^2),x, algorithm="giac")
```

```
[Out] -3/16*log(cos(x) + 1)/a + 3/16*log(-cos(x) + 1)/a + 1/8*(3*cos(x)^3 - 5*cos
(x))/((cos(x)^2 - 1)^2*a)
```

Mupad [B]
time = 0.08, size = 39, normalized size = 1.11

$$-\frac{3\,\mathrm{atanh}(\cos(x))}{8a} - \frac{\frac{5\cos(x)}{8} - \frac{3\cos(x)^3}{8}}{a\cos(x)^4 - 2a\cos(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a - a*cos(x)^2)),x)
```

```
[Out] - (3*atanh(cos(x)))/(8*a) - ((5*cos(x))/8 - (3*cos(x)^3)/8)/(a - 2*a*cos(x)
^2 + a*cos(x)^4)
```

3.10 $\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{(a+b)^3\mathrm{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{(a^2+3ab+3b^2)\cos(x)}{b^3}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}$$

```
[Out] (a^2+3*a*b+3*b^2)*cos(x)/b^3-1/3*(a+3*b)*cos(x)^3/b^2+1/5*cos(x)^5/b-(a+b)^
3*arctan(cos(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 398, 211}

$$\frac{(a^2+3ab+3b^2)\cos(x)}{b^3}-\frac{(a+b)^3\mathrm{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{(a+3b)\cos^3(x)}{3b^2}+\frac{\cos^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)^3*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2))) + ((a^2 +
 3*a*b + 3*b^2)*Cos[x])/b^3 - ((a + 3*b)*Cos[x]^3)/(3*b^2) + Cos[x]^5/(5*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^7(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{(1-x^2)^3}{a+bx^2}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-\frac{a^2+3ab+3b^2}{b^3} + \frac{(a+3b)x^2}{b^2} - \frac{x^4}{b} + \frac{a^3+3a^2b+3ab^2+b^3}{b^3(a+bx^2)}\right) dx, x, \text{c}$$
$$= \frac{(a^2+3ab+3b^2)\cos(x)}{b^3} - \frac{(a+3b)\cos^3(x)}{3b^2} + \frac{\cos^5(x)}{5b} - \frac{(a+b)^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x,\right.}{b^3}$$
$$= -\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}} + \frac{(a^2+3ab+3b^2)\cos(x)}{b^3} - \frac{(a+3b)\cos^3(x)}{3b^2} + \frac{\cos^5(x}{5b}$$

Mathematica [A]
time = 0.26, size = 143, normalized size = 1.83

$$-\frac{(a+b)^3\text{ArcTan}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}} - \frac{(a+b)^3\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}} + \frac{(8a^2+22ab+19b^2)\cos(x)}{8b^3} - \frac{(4a+9b)\cos(3x)}{48b^2} + \frac{\cos(5x)}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)^3*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/(Sqrt[a]*b^(
7/2))) - ((a + b)^3*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/(Sqrt
[a]*b^(7/2)) + ((8*a^2 + 22*a*b + 19*b^2)*Cos[x])/(8*b^3) - ((4*a + 9*b)*Co
s[3*x])/(48*b^2) + Cos[5*x]/(80*b)
```

Maple [A]
time = 0.23, size = 94, normalized size = 1.21

method	result
default	$\frac{\frac{\left(\cos^5(x)\right)b^2}{5} - \frac{ab\left(\cos^3(x)\right)}{3} - b^2\left(\cos^3(x)\right) + a^2\cos(x) + 3ab\cos(x) + 3b^2\cos(x)}{b^3} + \frac{\left(-a^3-3a^2b-3b^2a-b^3\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{e^{ix}a^2}{2b^3} + \frac{11a\,e^{ix}}{8b^2} + \frac{19\,e^{ix}}{16b} + \frac{e^{-ix}a^2}{2b^3} + \frac{11\,e^{-ix}a}{8b^2} + \frac{19\,e^{-ix}}{16b} - \frac{3i\ln\left(e^{2ix} + \frac{2ia\,e^{ix}}{\sqrt{ab}} + 1\right)a}{2\sqrt{ab}\,b} - \frac{3i\ln\left(e^{2ix} + \frac{2ia\,e^{ix}}{\sqrt{ab}} + 1\right)a^2}{2\sqrt{ab}\,b^2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*cos(x)^5*b^2-1/3*a*b*cos(x)^3-b^2*cos(x)^3+a^2*cos(x)+3*a*b*cos(
x)+3*b^2*cos(x))+(-a^3-3*a^2*b-3*a*b^2-b^3)/b^3/(a*b)^(1/2)*arctan(b*cos(x)
/(a*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 87, normalized size = 1.12

$$-\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{2}\cos\left(x\right)^{5}-5\,\left(ab+3\,b^{2}\right)\cos\left(x\right)^{3}+15\,\left(a^{2}+3\,ab+3\,b^{2}\right)\cos\left(x\right)}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*b^2*cos(x)^5 - 5*(a*b + 3*b^2)*cos(x)^3 + 15*(a^2 + 3*a*b + 3*b^
2)*cos(x))/b^3
```

Fricas [A]
time = 0.41, size = 225, normalized size = 2.88

$$\left[\frac{6\,ab^{3}\cos\left(x\right)^{5}-10\,\left(a^{2}b^{2}+3\,ab^{3}\right)\cos\left(x\right)^{3}-15\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^{2}+2\,\sqrt{-ab}\,\cos(x)-a}{b\cos(x)^{2}+a}\right)+30\,\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}\right)\cos\left(x\right)}{30\,ab^{4}},\ \frac{3\,ab^{3}\cos\left(x\right)^{5}-5\,\left(a^{2}b^{2}+3\,ab^{3}\right)\cos\left(x\right)^{3}-15\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,\cos(x)}{a}\right)+15\,\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}\right)\cos\left(x\right)}{15\,ab^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*cos(x)^5 - 10*(a^2*b^2 + 3*a*b^3)*cos(x)^3 - 15*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos(x) - a)/
(b*cos(x)^2 + a)) + 30*(a^3*b + 3*a^2*b^2 + 3*a*b^3)*cos(x))/(a*b^4), 1/15*
(3*a*b^3*cos(x)^5 - 5*(a^2*b^2 + 3*a*b^3)*cos(x)^3 - 15*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a*b)*cos(x)/a) + 15*(a^3*b + 3*a^2*b^2 +
 3*a*b^3)*cos(x))/(a*b^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**7/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 99, normalized size = 1.27

$$-\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{4}\cos\left(x\right)^{5}-5\,ab^{3}\cos\left(x\right)^{3}-15\,b^{4}\cos\left(x\right)^{3}+15\,a^{2}b^{2}\cos\left(x\right)+45\,ab^{3}\cos\left(x\right)+45\,b^{4}\cos\left(x\right)}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*b^4*cos(x)^5 - 5*a*b^3*cos(x)^3 - 15*b^4*cos(x)^3 + 15*a^2*b^2*c
os(x) + 45*a*b^3*cos(x) + 45*b^4*cos(x))/b^5
```

Mupad [B]
time = 2.13, size = 100, normalized size = 1.28

$$\cos(x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}+\frac{3}{b}\right)}{b}\right)-\cos(x)^3\left(\frac{a}{3\,b^2}+\frac{1}{b}\right)+\frac{\cos(x)^5}{5\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\cos(x)\,(a+b)^3}{\sqrt{a}\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}\right)(a+b)^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/(a + b*cos(x)^2),x)
```

```
[Out] cos(x)*(3/b + (a*(a/b^2 + 3/b))/b) - cos(x)^3*(a/(3*b^2) + 1/b) + cos(x)^5/
(5*b) - (atan((b^(1/2)*cos(x)*(a + b)^3)/(a^(1/2)*(3*a*b^2 + 3*a^2*b + a^3
+ b^3)))*(a + b)^3)/(a^(1/2)*b^(7/2))
```

3.11 $\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}$$

```
[Out] (a+2*b)*cos(x)/b^2-1/3*cos(x)^3/b-(a+b)^2*arctan(cos(x)*b^(1/2)/a^(1/2))/b^
(5/2)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3269, 398, 211}

$$-\frac{(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)^2*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2))) + ((a + 2
*b)*Cos[x])/b^2 - Cos[x]^3/(3*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^2}\,dx,x,\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2\left(a+bx^2\right)}\right)\,dx,x,\cos(x)\right)\\
&= \frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}-\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\cos(x)\right)}{b^2}\\
&= -\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{(a+2b)\cos(x)}{b^2}-\frac{\cos^3(x)}{3b}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

time = 0.19, size = 116, normalized size = 2.15

$$\frac{-\frac{12(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{12(a+b)^2\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)}{\sqrt{a}}+3\sqrt{b}\,(4a+7b)\cos(x)-b^{3/2}\cos(3x)}{12b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] ((-12*(a + b)^2*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/Sqrt[a] -
 (12*(a + b)^2*ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]])/Sqrt[a] +
3*Sqrt[b]*(4*a + 7*b)*Cos[x] - b^(3/2)*Cos[3*x])/(12*b^(5/2))
```

Maple [**A**]

time = 0.16, size = 57, normalized size = 1.06

method	result
default	$\frac{-\frac{b\left(\cos^3(x)\right)}{3}+a\cos(x)+2b\cos(x)}{b^2}+\frac{\left(-a^2-2ab-b^2\right)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{a\,e^{ix}}{2b^2}+\frac{7\,e^{ix}}{8b}+\frac{e^{-ix}a}{2b^2}+\frac{7\,e^{-ix}}{8b}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a^2}{2\sqrt{ab}\,b^2}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a}{\sqrt{ab}\,b}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}+ i\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/3*b*cos(x)^3+a*cos(x)+2*b*cos(x))+(-a^2-2*a*b-b^2)/b^2/(a*b)^(1/2
)*arctan(b*cos(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 53, normalized size = 0.98

$$-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{b\cos\left(x\right)^3-3\,(a+2\,b)\cos\left(x\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -(a^2 + 2*a*b + b^2)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/3*(b*co
s(x)^3 - 3*(a + 2*b)*cos(x))/b^2
```

Fricas [A]
time = 0.43, size = 152, normalized size = 2.81

$$\left[-\frac{2\,ab^2\cos\left(x\right)^3+3\,(a^2+2\,ab+b^2)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\,\cos(x)-a}{b\cos(x)^2+a}\right)-6\,(a^2b+2\,ab^2)\cos\left(x\right)}{6\,ab^3},-\frac{ab^2\cos\left(x\right)^3+3\,(a^2+2\,ab+b^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,\cos(x)}{a}\right)-3\,(a^2b+2\,ab^2)\cos\left(x\right)}{3\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a*b^2*cos(x)^3 + 3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2
 + 2*sqrt(-a*b)*cos(x) - a)/(b*cos(x)^2 + a)) - 6*(a^2*b + 2*a*b^2)*cos(x))
/(a*b^3), -1/3*(a*b^2*cos(x)^3 + 3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(sqr
t(a*b)*cos(x)/a) - 3*(a^2*b + 2*a*b^2)*cos(x))/(a*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 59, normalized size = 1.09

$$-\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}-\frac{b^2\cos\left(x\right)^3-3\,ab\cos\left(x\right)-6\,b^2\cos\left(x\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(a^2 + 2*a*b + b^2)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/3*(b^2*
cos(x)^3 - 3*a*b*cos(x) - 6*b^2*cos(x))/b^3
```

Mupad [B]
time = 0.10, size = 65, normalized size = 1.20

$$\cos(x)\left(\frac{a}{b^2}+\frac{2}{b}\right)-\frac{\cos(x)^3}{3\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\cos(x)\,(a+b)^2}{\sqrt{a}\,(a^2+2\,a\,b+b^2)}\right)\,(a+b)^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(a + b*cos(x)^2),x)
```

```
[Out] cos(x)*(a/b^2 + 2/b) - cos(x)^3/(3*b) - (atan((b^(1/2)*cos(x)*(a + b)^2)/(a
^(1/2)*(2*a*b + a^2 + b^2)))*(a + b)^2)/(a^(1/2)*b^(5/2))
```

3.12 $\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{\cos(x)}{b}$$

```
[Out] cos(x)/b-(a+b)*arctan(cos(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3269, 396, 211}

$$\frac{\cos(x)}{b}-\frac{(a+b)\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -(((a + b)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + Cos[x]/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx, x, \cos(x)\right)$$
$$= \frac{\cos(x)}{b} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)}{b}$$
$$= -\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} + \frac{\cos(x)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(36) = 72.

time = 0.18, size = 90, normalized size = 2.50

$$\frac{-\left((a+b)\text{ArcTan}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)\right)-(a+b)\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+\sqrt{a}\sqrt{b}\cos(x)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] (-((a + b)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]]) - (a + b)*ArcT
an[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*Sqrt[b]*Cos[x])/(Sqr
t[a]*b^(3/2))
```

Maple [A]

time = 0.13, size = 34, normalized size = 0.94

method	result
default	$\frac{\cos(x)}{b}+\frac{(-a-b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{b\sqrt{ab}}$
risch	$\frac{e^{ix}}{2b}+\frac{e^{-ix}}{2b}+\frac{i\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a}{2\sqrt{ab}\,b}+\frac{i\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)a}{2\sqrt{ab}\,b}-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)/b+(-a-b)/b/(a*b)^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))
```

Maxima [A]

time = 0.47, size = 30, normalized size = 0.83

$$-\frac{(a+b)\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{\cos(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -(a + b)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b) + cos(x)/b
```

Fricas [A]
time = 0.40, size = 95, normalized size = 2.64

$$\left[\frac{2\,ab\cos\left(x\right)-\sqrt{-ab}\,\left(a+b\right)\log\left(-\frac{b\cos\left(x\right)^{2}+2\,\sqrt{-ab}\,\cos\left(x\right)-a}{b\cos\left(x\right)^{2}+a}\right)}{2\,ab^{2}},\frac{ab\cos\left(x\right)-\sqrt{ab}\,\left(a+b\right)\arctan\left(\frac{\sqrt{ab}\,\cos\left(x\right)}{a}\right)}{ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*cos(x) - sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos
(x) - a)/(b*cos(x)^2 + a)))/(a*b^2), (a*b*cos(x) - sqrt(a*b)*(a + b)*arctan
(sqrt(a*b)*cos(x)/a))/(a*b^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 30, normalized size = 0.83

$$-\frac{\left(a+b\right)\arctan\left(\frac{b\cos\left(x\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{\cos\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(a + b)*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*b) + cos(x)/b
```

Mupad [B]
time = 0.09, size = 28, normalized size = 0.78

$$\frac{\cos\left(x\right)}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\cos\left(x\right)}{\sqrt{a}}\right)\,\left(a+b\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a + b*cos(x)^2),x)
[Out] cos(x)/b - (atan((b^(1/2)*cos(x))/a^(1/2))*(a + b))/(a^(1/2)*b^(3/2))
```

3.13 $\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=26

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] -arctan(cos(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3269, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Cos[x]^2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.00

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x]^2),x]
[Out] -(ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]))
```

Maple [A]
time = 0.06, size = 18, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	18
default	$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$	18
risch	$-\frac{i\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}+\frac{i\ln\left(e^{2ix}-\frac{2ia\,e^{ix}}{\sqrt{ab}}+1\right)}{2\sqrt{ab}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/(a*b)^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.65

$$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="maxima")
[Out] -arctan(b*cos(x)/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.41, size = 73, normalized size = 2.81

$$\left[-\frac{\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\,\cos(x)-a}{b\cos(x)^2+a}\right)}{2\,ab},\,-\frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,\cos(x)}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*cos(x) - a)/(b*cos(x)^2 +
a))/(a*b), -sqrt(a*b)*arctan(sqrt(a*b)*cos(x)/a)/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.41, size = 66, normalized size = 2.54

$$\begin{cases} \frac{\tilde{\infty}}{\cos(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{b\cos(x)} & \text{for } a = 0 \\ -\frac{\cos(x)}{a} & \text{for } b = 0 \\ -\frac{\log\left(-\sqrt{-\frac{a}{b}}+\cos(x)\right)}{2b\sqrt{-\frac{a}{b}}} + \frac{\log\left(\sqrt{-\frac{a}{b}}+\cos(x)\right)}{2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)**2),x)
```

```
[Out] Piecewise((zoo/cos(x), Eq(a, 0) & Eq(b, 0)), (1/(b*cos(x)), Eq(a, 0)), (-co
s(x)/a, Eq(b, 0)), (-log(-sqrt(-a/b) + cos(x))/(2*b*sqrt(-a/b)) + log(sqrt(
-a/b) + cos(x))/(2*b*sqrt(-a/b)), True))
```

Giac [A]
time = 0.41, size = 17, normalized size = 0.65

$$-\frac{\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(b*cos(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 2.23, size = 18, normalized size = 0.69

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x)^2),x)
```

```
[Out] -atan((b^(1/2)*cos(x))/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.14 $\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}-\frac{\tanh^{-1}(\cos(x))}{a+b}$$

```
[Out] -arctanh(cos(x))/(a+b)-arctan(cos(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3269, 400, 212, 211}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}-\frac{\tanh^{-1}(\cos(x))}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Cos[x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b))) - ArcTanh[C
os[x]]/(a + b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
```

```
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+bx^2)}\,dx, x, \cos(x)\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)}{a+b} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)}{a+b}\\ &= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)} - \frac{\tanh^{-1}(\cos(x))}{a+b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 1.19

$$\frac{-\frac{2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}} + \log(1-\cos(x)) - \log(1+\cos(x))}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ((-2*Sqrt[b]*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/Sqrt[a] + Log[1 - Cos[x]] -
Log[1 + Cos[x]])/(2*(a + b))
```

Maple [A]
time = 0.13, size = 56, normalized size = 1.33

method	result	size
default	$-\frac{\ln(\cos(x)+1)}{2a+2b}+\frac{\ln(-1+\cos(x))}{2a+2b}-\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}$	56
risch	$-\frac{\ln(e^{ix}+1)}{a+b}+\frac{\ln(e^{ix}-1)}{a+b}+\frac{i\sqrt{ab}\,\ln\left(e^{2ix}-\frac{2i\sqrt{ab}\,e^{ix}}{b}+1\right)}{2a(a+b)}-\frac{i\sqrt{ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{ab}\,e^{ix}}{b}+1\right)}{2a(a+b)}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(2*a+2*b)*ln(cos(x)+1)+1/(2*a+2*b)*ln(-1+cos(x))-b/(a+b)/(a*b)^(1/2)*arc
tan(b*cos(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 48, normalized size = 1.14

$$-\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}-\frac{\log\left(\cos\left(x\right)+1\right)}{2\,(a+b)}+\frac{\log\left(\cos\left(x\right)-1\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*log(cos(x) + 1)/(a
+ b) + 1/2*log(cos(x) - 1)/(a + b)
```

Fricas [A]
time = 0.45, size = 113, normalized size = 2.69

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right)-\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{2\,(a+b)},-\frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\cos\left(x\right)\right)+\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{2\,(a+b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x) - a)/(b*cos(x)^2 +
 a)) - log(1/2*cos(x) + 1/2) + log(-1/2*cos(x) + 1/2))/(a + b), -1/2*(2*sqr
t(b/a)*arctan(sqrt(b/a)*cos(x)) + log(1/2*cos(x) + 1/2) - log(-1/2*cos(x) +
 1/2))/(a + b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{a+b\cos^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.42, size = 50, normalized size = 1.19

$$-\frac{b\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}-\frac{\log\left(\cos\left(x\right)+1\right)}{2\,(a+b)}+\frac{\log\left(-\cos\left(x\right)+1\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(b*cos(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/2*log(cos(x) + 1)/(a
+ b) + 1/2*log(-cos(x) + 1)/(a + b)
```

Mupad [B]
time = 2.67, size = 853, normalized size = 20.31

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}+4\,b^3\,\cos(x)\right)1\mathrm{i}}{2\,(a+b)}-\frac{\left(\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}-4\,b^3\,\cos(x)\right)1\mathrm{i}}{2\,(a+b)}}{\frac{\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}+4\,b^3\,\cos(x)}{2\,(a+b)}+\frac{\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\cos(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}-4\,b^3\,\cos(x)}{2\,(a+b)}}\right)1\mathrm{i}}{a+b}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\cos(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)1\mathrm{i}}{a^2+b\,a}+\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2+\frac{\cos(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)1\mathrm{i}}{a^2+b\,a}}{\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)+\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2-\frac{\cos(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)}{a^2+b\,a}-\frac{\sqrt{-a\,b}\,\left(2\,b^3\,\cos(x)-\frac{\sqrt{-a\,b}\,\left(4\,a\,b^3+2\,b^4+2\,a^2\,b^2+\frac{\cos(x)\,\sqrt{-a\,b}\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{4\,\left(a^2+b\,a\right)}\right)}{2\,\left(a^2+b\,a\right)}\right)}{a^2+b\,a}}\right)\sqrt{-a\,b}\,1\mathrm{i}}{a\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + b*cos(x)^2)),x)
```

```
[Out] (atan(((((8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^
3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)) + 4*b^3*cos(x))*1i)/(2*(a + b)) -
(((8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a
^3*b^2))/(2*(a + b)))/(2*(a + b)) - 4*b^3*cos(x))*1i)/(2*(a + b)))/(((8*a*b
^3 + 4*b^4 + 4*a^2*b^2 - (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))
/(2*(a + b)))/(2*(a + b)) + 4*b^3*cos(x))/(2*(a + b)) + ((8*a*b^3 + 4*b^4 +
 4*a^2*b^2 + (cos(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b))
)/(2*(a + b)) - 4*b^3*cos(x))/(2*(a + b))))*1i)/(a + b) + (atan((((-a*b)^(1
/2)*(2*b^3*cos(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (cos(x)*(-
a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*
(a*b + a^2)))*1i)/(a*b + a^2) + ((-a*b)^(1/2)*(2*b^3*cos(x) - ((-a*b)^(1/2)
*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a
^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*1i)/(a*b + a^2))/((
(-a*b)^(1/2)*(2*b^3*cos(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (
cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^
2))))/(2*(a*b + a^2))))/(a*b + a^2) - ((-a*b)^(1/2)*(2*b^3*cos(x) - ((-a*b)
^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 + (cos(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5
 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2))))/(a*b + a^2))
)*(-a*b)^(1/2)*1i)/(a*(a + b))
```

3.15 $\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=62

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}-\frac{(a+3b)\tanh^{-1}(\cos(x))}{2(a+b)^2}-\frac{\cot(x)\csc(x)}{2(a+b)}$$

```
[Out] -1/2*(a+3*b)*arctanh(cos(x))/(a+b)^2-1/2*cot(x)*csc(x)/(a+b)-b^(3/2)*arctan
(cos(x)*b^(1/2)/a^(1/2))/(a+b)^2/a^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3269, 425, 536, 212, 211}

$$-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}-\frac{(a+3b)\tanh^{-1}(\cos(x))}{2(a+b)^2}-\frac{\cot(x)\csc(x)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2)) - ((a + 3
*b)*ArcTanh[Cos[x]])/(2*(a + b)^2) - (Cot[x]*Csc[x])/(2*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(1-x^2)^2\,(a+bx^2)}\,dx, x, \cos(x)\right) \\
&= -\frac{\cot(x)\csc(x)}{2(a+b)} - \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \cos(x)\right)}{2(a+b)} \\
&= -\frac{\cot(x)\csc(x)}{2(a+b)} - \frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)}{(a+b)^2} - \frac{(a+3b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x\right.}{2(a+b)^2} \\
&= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2} - \frac{(a+3b)\tanh^{-1}(\cos(x))}{2(a+b)^2} - \frac{\cot(x)\csc(x)}{2(a+b)}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.
time = 0.57, size = 140, normalized size = 2.26

$$\frac{-8b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)-8b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+\sqrt{a}\left(-\left((a+b)\csc^2\left(\frac{x}{2}\right)\right)-4(a+3b)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+(a+b)\sec^2\left(\frac{x}{2}\right)\right)}{8\sqrt{a}\,(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] (-8*b^(3/2)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]] - 8*b^(3/2)*Ar
cTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*(-((a + b)*Csc[x/2
```

```
]^2) - 4*(a + 3*b)*(Log[Cos[x/2]] - Log[Sin[x/2]]) + (a + b)*Sec[x/2]^2))/(
8*Sqrt[a]*(a + b)^2)
```

Maple [A]
time = 0.21, size = 95, normalized size = 1.53

method	result
default	$\frac{1}{(4a+4b)(\cos(x)+1)}+\frac{(-a-3b)\ln(\cos(x)+1)}{4(a+b)^2}+\frac{1}{(4a+4b)(-1+\cos(x))}+\frac{(a+3b)\ln(-1+\cos(x))}{4(a+b)^2}-\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}$
risch	$\frac{e^{3ix}+e^{ix}}{(e^{2ix}-1)^2(a+b)}+\frac{\ln(e^{ix}-1)a}{2a^2+4ab+2b^2}+\frac{3\ln(e^{ix}-1)b}{2(a^2+2ab+b^2)}-\frac{\ln(e^{ix}+1)a}{2(a^2+2ab+b^2)}-\frac{3\ln(e^{ix}+1)b}{2(a^2+2ab+b^2)}-\frac{i\sqrt{ab}\,b\ln\left(e^{2ix}+\frac{2i\sqrt{ab}\,e^{ix}}{b}+1\right)}{2a(a+b)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4*a+4*b)/(cos(x)+1)+1/4/(a+b)^2*(-a-3*b)*ln(cos(x)+1)+1/(4*a+4*b)/(-1+co
s(x))+1/4*(a+3*b)/(a+b)^2*ln(-1+cos(x))-b^2/(a+b)^2/(a*b)^(1/2)*arctan(b*co
s(x)/(a*b)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(50) = 100.
time = 0.48, size = 105, normalized size = 1.69

$$-\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}-\frac{(a+3\,b)\log(\cos(x)+1)}{4\,(a^2+2\,ab+b^2)}+\frac{(a+3\,b)\log(\cos(x)-1)}{4\,(a^2+2\,ab+b^2)}+\frac{\cos(x)}{2\,((a+b)\cos(x)^2-a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b^2*arctan(b*cos(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 1/4*(a +
3*b)*log(cos(x) + 1)/(a^2 + 2*a*b + b^2) + 1/4*(a + 3*b)*log(cos(x) - 1)/(a
^2 + 2*a*b + b^2) + 1/2*cos(x)/((a + b)*cos(x)^2 - a - b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(50) = 100.
time = 0.46, size = 274, normalized size = 4.42

$$\left[\frac{2\,(b\cos(x)^2-b)\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right)+2\,(a+b)\cos(x)-((a+3\,b)\cos(x)^2-a-3\,b)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+((a+3\,b)\cos(x)^2-a-3\,b)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{4\,((a^2+2\,ab+b^2)\cos(x)^2-a^2-2\,ab-b^2)},-\frac{4\,(b\cos(x)^2-b)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\cos(x)\right)-2\,(a+b)\cos(x)+((a+3\,b)\cos(x)^2-a-3\,b)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-((a+3\,b)\cos(x)^2-a-3\,b)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{4\,((a^2+2\,ab+b^2)\cos(x)^2-a^2-2\,ab-b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*cos(x)^2 - b)*sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x)
 - a)/(b*cos(x)^2 + a)) + 2*(a + b)*cos(x) - ((a + 3*b)*cos(x)^2 - a - 3*b)
*log(1/2*cos(x) + 1/2) + ((a + 3*b)*cos(x)^2 - a - 3*b)*log(-1/2*cos(x) + 1
/2))/((a^2 + 2*a*b + b^2)*cos(x)^2 - a^2 - 2*a*b - b^2), -1/4*(4*(b*cos(x)^
2 - b)*sqrt(b/a)*arctan(sqrt(b/a)*cos(x)) - 2*(a + b)*cos(x) + ((a + 3*b)*c
os(x)^2 - a - 3*b)*log(1/2*cos(x) + 1/2) - ((a + 3*b)*cos(x)^2 - a - 3*b)*l
og(-1/2*cos(x) + 1/2))/((a^2 + 2*a*b + b^2)*cos(x)^2 - a^2 - 2*a*b - b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**3/(a + b*cos(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(50) = 100.
time = 0.43, size = 103, normalized size = 1.66

$$-\frac{b^2\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{(a+3\,b)\log(\cos(x)+1)}{4\,(a^2+2\,ab+b^2)} + \frac{(a+3\,b)\log(-\cos(x)+1)}{4\,(a^2+2\,ab+b^2)} + \frac{\cos(x)}{2\left(\cos(x)^2-1\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(b*cos(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 1/4*(a +
3*b)*log(cos(x) + 1)/(a^2 + 2*a*b + b^2) + 1/4*(a + 3*b)*log(-cos(x) + 1)/(
a^2 + 2*a*b + b^2) + 1/2*cos(x)/((cos(x)^2 - 1)*(a + b))
```

Mupad [B]
time = 2.59, size = 1138, normalized size = 18.35

$$\ln(\cos(x)-1)\left(\frac{b}{2(a+b)^2}+\frac{1}{4(a+b)}\right)-\frac{\cos(x)}{2\sin(x)^2(a+b)}-\frac{\ln(\cos(x)+1)(a+3b)}{4(a+b)^2}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-ab^3}\left(\frac{\cos(x)(a^2b^3+6ab^4+13b^5)}{4(a^2+2ab+b^2)}+\frac{\left(\frac{2a^5b^2+12a^4b^3+28a^3b^4+32a^2b^5+18ab^6+4b^7}{2(a^3+3a^2b+3ab^2+b^3)}-\frac{\cos(x)\sqrt{-ab^3}(-16a^5b^2-48a^4b^3-32a^3b^4+32a^2b^5+48ab^6+16b^7)}{8(a^2+2ab+b^2)(a^3+2a^2b+ab^2)}\right)\sqrt{-ab^3}}{2(a^3+2a^2b+ab^2)}\right)1i}{a^3+2a^2b+ab^2}+\frac{\sqrt{-ab^3}\left(\frac{\cos(x)(a^2b^3+6ab^4+13b^5)}{4(a^2+2ab+b^2)}-\frac{\left(\frac{2a^5b^2+12a^4b^3+28a^3b^4+32a^2b^5+18ab^6+4b^7}{2(a^3+3a^2b+3ab^2+b^3)}+\frac{\cos(x)\sqrt{-ab^3}(-16a^5b^2-48a^4b^3-32a^3b^4+32a^2b^5+48ab^6+16b^7)}{8(a^2+2ab+b^2)(a^3+2a^2b+ab^2)}\right)\sqrt{-ab^3}}{2(a^3+2a^2b+ab^2)}\right)1i}{a^3+2a^2b+ab^2}}{\frac{\frac{3b^5}{2}+\frac{ab^4}{2}}{a^3+3a^2b+3ab^2+b^3}-\frac{\sqrt{-ab^3}\left(\frac{\cos(x)(a^2b^3+6ab^4+13b^5)}{4(a^2+2ab+b^2)}+\frac{\left(\frac{2a^5b^2+12a^4b^3+28a^3b^4+32a^2b^5+18ab^6+4b^7}{2(a^3+3a^2b+3ab^2+b^3)}-\frac{\cos(x)\sqrt{-ab^3}(-16a^5b^2-48a^4b^3-32a^3b^4+32a^2b^5+48ab^6+16b^7)}{8(a^2+2ab+b^2)(a^3+2a^2b+ab^2)}\right)\sqrt{-ab^3}}{2(a^3+2a^2b+ab^2)}\right)}{a^3+2a^2b+ab^2}+\frac{\sqrt{-ab^3}\left(\frac{\cos(x)(a^2b^3+6ab^4+13b^5)}{4(a^2+2ab+b^2)}-\frac{\left(\frac{2a^5b^2+12a^4b^3+28a^3b^4+32a^2b^5+18ab^6+4b^7}{2(a^3+3a^2b+3ab^2+b^3)}+\frac{\cos(x)\sqrt{-ab^3}(-16a^5b^2-48a^4b^3-32a^3b^4+32a^2b^5+48ab^6+16b^7)}{8(a^2+2ab+b^2)(a^3+2a^2b+ab^2)}\right)\sqrt{-ab^3}}{2(a^3+2a^2b+ab^2)}\right)}{a^3+2a^2b+ab^2}}\right)\sqrt{-ab^3}\,1i}{a^3+2a^2b+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a + b*cos(x)^2)),x)
```

```
[Out] log(cos(x) - 1)*(b/(2*(a + b)^2) + 1/(4*(a + b))) - cos(x)/(2*sin(x)^2*(a +
 b)) - (log(cos(x) + 1)*(a + 3*b))/(4*(a + b)^2) - (atan((((-a*b^3)^(1/2)*(
(cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6
```

```
 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 +
3*a^2*b + a^3 + b^3)) - (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*
b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2
+ 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2
+ 2*a^2*b + a^3) + ((-a*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(
4*(2*a*b + a^2 + b^2)) - (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12
*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (cos(x)*(-a*b^3
)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*
b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(
a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2 + 2*a^2*b + a^3))/(((a*b^4)/2 + (3*b^5)
/2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - ((-a*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 1
3*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6 + 4*b^7 + 32*a^2*b^
5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
) - (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 4
8*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(
-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))))/(a*b^2 + 2*a^2*b + a^3) + ((-a
*b^3)^(1/2)*((cos(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2))
- (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(
2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (cos(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*
b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 +
 b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))
))/(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2)*1i)/(a*b^2 + 2*a^2*b + a^3)
```

3.16 $\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=94

$$-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}-\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\cos(x))}{8(a+b)^3}-\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2}-\frac{\cot(x)\csc^3(x)}{4(a+b)}$$

```
[Out] -1/8*(3*a^2+10*a*b+15*b^2)*arctanh(cos(x))/(a+b)^3-1/8*(3*a+7*b)*cot(x)*csc
(x)/(a+b)^2-1/4*cot(x)*csc(x)^3/(a+b)-b^(5/2)*arctan(cos(x)*b^(1/2)/a^(1/2)
)/(a+b)^3/a^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3269, 425, 541, 536, 212, 211}

$$-\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\cos(x))}{8(a+b)^3}-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b}\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}-\frac{\cot(x)\csc^3(x)}{4(a+b)}-\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cos[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3)) - ((3*a^2
 + 10*a*b + 15*b^2)*ArcTanh[Cos[x]])/(8*(a + b)^3) - ((3*a + 7*b)*Cot[x]*Cs
c[x])/(8*(a + b)^2) - (Cot[x]*Csc[x]^3)/(4*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(1-x^2)^3\,(a+bx^2)}\,dx, x, \cos(x)\right) \\
&= -\frac{\cot(x)\csc^3(x)}{4(a+b)} - \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{(1-x^2)^2(a+bx^2)}\,dx, x, \cos(x)\right)}{4(a+b)} \\
&= -\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2} - \frac{\cot(x)\csc^3(x)}{4(a+b)} - \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{(1-x^2)(a+bx^2)}\,dx, x, \cos(
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(3a+7b)\cot(x)\csc(x)}{8(a+b)^2} - \frac{\cot(x)\csc^3(x)}{4(a+b)} - \frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \cos(x)\right)}{(a+b)^3} - \frac{(3a^2+
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\cos(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3} - \frac{(3a^2+10ab+15b^2)\tanh^{-1}(\cos(x))}{8(a+b)^3} - \frac{(3a+7b)\cot(x)\csc
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 204 vs. 2(94) = 188.
time = 1.50, size = 204, normalized size = 2.17

$$\frac{-64b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}-\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)-64b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}+\sqrt{a+b}\tan\left(\frac{x}{2}\right)}{\sqrt{a}}\right)+\sqrt{a}\left(-2(3a^2+10ab+7b^2)\csc^2\left(\frac{x}{2}\right)-(a+b)^2\csc^4\left(\frac{x}{2}\right)-8(3a^2+10ab+15b^2)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+2(3a^2+10ab+7b^2)\sec^2\left(\frac{x}{2}\right)+(a+b)^2\sec^4\left(\frac{x}{2}\right)\right)}{64\sqrt{a}\,(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] (-64*b^(5/2)*ArcTan[(Sqrt[b] - Sqrt[a + b]*Tan[x/2])/Sqrt[a]] - 64*b^(5/2)*
ArcTan[(Sqrt[b] + Sqrt[a + b]*Tan[x/2])/Sqrt[a]] + Sqrt[a]*(-2*(3*a^2 + 10*
a*b + 7*b^2)*Csc[x/2]^2 - (a + b)^2*Csc[x/2]^4 - 8*(3*a^2 + 10*a*b + 15*b^2
)*(Log[Cos[x/2]] - Log[Sin[x/2]]) + 2*(3*a^2 + 10*a*b + 7*b^2)*Sec[x/2]^2 +
 (a + b)^2*Sec[x/2]^4))/(64*Sqrt[a]*(a + b)^3)
```

Maple [A]
time = 0.25, size = 155, normalized size = 1.65

method	result
default	$\frac{1}{2(8a+8b)(\cos(x)+1)^2}-\frac{-3a-7b}{16(a+b)^2(\cos(x)+1)}+\frac{(-3a^2-10ab-15b^2)\ln(\cos(x)+1)}{16(a+b)^3}-\frac{1}{2(8a+8b)(-1+\cos(x))^2}-\frac{-3a-}{16(a+b)^2(-1}$
risch	$\frac{3a\,e^{7ix}+7b\,e^{7ix}-11a\,e^{5ix}-15b\,e^{5ix}-11a\,e^{3ix}-15b\,e^{3ix}+3a\,e^{ix}+7b\,e^{ix}}{4(a+b)^2(e^{2ix}-1)^4}+\frac{3\ln(e^{ix}-1)a^2}{8(a^3+3a^2b+3b^2a+b^3)}+\frac{5\ln(e^{ix}-1)ab}{4(a^3+3a^2b+3b^2a+b^3)}+\frac{}{8(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(8*a+8*b)/(cos(x)+1)^2-1/16*(-3*a-7*b)/(a+b)^2/(cos(x)+1)+1/16/(a+b)^3*
(-3*a^2-10*a*b-15*b^2)*ln(cos(x)+1)-1/2/(8*a+8*b)/(-1+cos(x))^2-1/16*(-3*a-
7*b)/(a+b)^2/(-1+cos(x))+1/16*(3*a^2+10*a*b+15*b^2)/(a+b)^3*ln(-1+cos(x))-b
^3/(a+b)^3/(a*b)^(1/2)*arctan(b*cos(x)/(a*b)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(80) = 160.
time = 0.47, size = 200, normalized size = 2.13

$$-\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}}-\frac{(3a^2+10ab+15b^2)\log(\cos(x)+1)}{16(a^3+3a^2b+3ab^2+b^3)}+\frac{(3a^2+10ab+15b^2)\log(\cos(x)-1)}{16(a^3+3a^2b+3ab^2+b^3)}+\frac{(3a+7b)\cos(x)^3-(5a+9b)\cos(x)}{8\left((a^2+2ab+b^2)\cos(x)^4-2(a^2+2ab+b^2)\cos(x)^2+a^2+2ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*cos(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
 - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2
```

```
+ b^3) + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) - 1)/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3) + 1/8*((3*a + 7*b)*cos(x)^3 - (5*a + 9*b)*cos(x))/((a^2 + 2*a*
b + b^2)*cos(x)^4 - 2*(a^2 + 2*a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(80) = 160.
time = 0.49, size = 592, normalized size = 6.30

$$\left[\frac{2\,(3\,a^2+10\,ab+7\,b^2)\cos(x)^3+8\,(b^2\cos(x)^4-2\,b^2\cos(x)^2+b^2)\sqrt{-\frac{b}{a}}\log\left(\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\cos(x)-a}{b\cos(x)^2+a}\right)-2\,(5\,a^2+14\,ab+9\,b^2)\cos(x)-((3\,a^2+10\,ab+15\,b^2)\cos(x)^4-2\,(3\,a^2+10\,ab+15\,b^2)\cos(x)^2+3\,a^2+10\,ab+15\,b^2)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+((3\,a^2+10\,ab+15\,b^2)\cos(x)^4-2\,(3\,a^2+10\,ab+15\,b^2)\cos(x)^2+3\,a^2+10\,ab+15\,b^2)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{16\,((a^3+3\,a^2b+3\,ab^2+b^3)\cos(x)^4+a^3+3\,a^2b+3\,ab^2+b^3-2\,(a^3+3\,a^2b+3\,ab^2+b^3)\cos(x)^2)},\ \frac{2\,(3\,a^2+10\,ab+7\,b^2)\cos(x)^3-16\,(b^2\cos(x)^4-2\,b^2\cos(x)^2+b^2)\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\cos(x)\right)-2\,(5\,a^2+14\,ab+9\,b^2)\cos(x)-((3\,a^2+10\,ab+15\,b^2)\cos(x)^4-2\,(3\,a^2+10\,ab+15\,b^2)\cos(x)^2+3\,a^2+10\,ab+15\,b^2)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+((3\,a^2+10\,ab+15\,b^2)\cos(x)^4-2\,(3\,a^2+10\,ab+15\,b^2)\cos(x)^2+3\,a^2+10\,ab+15\,b^2)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{16\,((a^3+3\,a^2b+3\,ab^2+b^3)\cos(x)^4+a^3+3\,a^2b+3\,ab^2+b^3-2\,(a^3+3\,a^2b+3\,ab^2+b^3)\cos(x)^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 + 10*a*b + 7*b^2)*cos(x)^3 + 8*(b^2*cos(x)^4 - 2*b^2*cos(x)
^2 + b^2)*sqrt(-b/a)*log((b*cos(x)^2 - 2*a*sqrt(-b/a)*cos(x) - a)/(b*cos(x)
^2 + a)) - 2*(5*a^2 + 14*a*b + 9*b^2)*cos(x) - ((3*a^2 + 10*a*b + 15*b^2)*c
os(x)^4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*l
og(1/2*cos(x) + 1/2) + ((3*a^2 + 10*a*b + 15*b^2)*cos(x)^4 - 2*(3*a^2 + 10*
a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*log(-1/2*cos(x) + 1/2))/(
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 -
2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2), 1/16*(2*(3*a^2 + 10*a*b + 7*b^
2)*cos(x)^3 - 16*(b^2*cos(x)^4 - 2*b^2*cos(x)^2 + b^2)*sqrt(b/a)*arctan(sqr
t(b/a)*cos(x)) - 2*(5*a^2 + 14*a*b + 9*b^2)*cos(x) - ((3*a^2 + 10*a*b + 15*
b^2)*cos(x)^4 - 2*(3*a^2 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*
b^2)*log(1/2*cos(x) + 1/2) + ((3*a^2 + 10*a*b + 15*b^2)*cos(x)^4 - 2*(3*a^2
 + 10*a*b + 15*b^2)*cos(x)^2 + 3*a^2 + 10*a*b + 15*b^2)*log(-1/2*cos(x) + 1
/2))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**5/(a + b*cos(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(80) = 160.
time = 0.40, size = 178, normalized size = 1.89

$$-\frac{b^3\arctan\left(\frac{b\cos(x)}{\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}-\frac{(3\,a^2+10\,ab+15\,b^2)\log(\cos(x)+1)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{(3\,a^2+10\,ab+15\,b^2)\log(-\cos(x)+1)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)}+\frac{3\,a\cos(x)^3+7\,b\cos(x)^3-5\,a\cos(x)-9\,b\cos(x)}{8\,(a^2+2\,ab+b^2)\left(\cos(x)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*cos(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
 - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(cos(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-cos(x) + 1)/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 1/8*(3*a*cos(x)^3 + 7*b*cos(x)^3 - 5*a*cos(x) - 9*b*cos(x))
/((a^2 + 2*a*b + b^2)*(cos(x)^2 - 1)^2)
```

Mupad [B]
time = 5.28, size = 833, normalized size = 8.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5*(a + b*cos(x)^2)),x)
```

```
[Out] -(atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a*b^5)^(3/2)*64i + a^6*b*c
os(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*cos(
x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*cos(x
)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5
 + 60*a^6*b^4 + 9*a^7*b^3))*(-a*b^5)^(1/2)*8i - 3*a^3*cos(x)^3 + 3*a^3*atan
h(cos(x)) + 5*a^3*cos(x) - atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a
*b^5)^(3/2)*64i + a^6*b*cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^
(1/2)*289i + a^3*b^4*cos(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(
1/2)*190i + a^5*b^2*cos(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 +
300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*cos(x)^2*(-a*b^5)^(1/2
)*16i + atan((a*cos(x)*(-a*b^5)^(3/2)*64i - b*cos(x)*(-a*b^5)^(3/2)*64i + a
^6*b*cos(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*cos(x)*(-a*b^5)^(1/2)*289i + a^3*b^
4*cos(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*cos(x)*(-a*b^5)^(1/2)*190i + a^5*b^2
*cos(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a
^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*cos(x)^4*(-a*b^5)^(1/2)*8i + 9*a*b^2*cos(
x) + 14*a^2*b*cos(x) - 6*a^3*atanh(cos(x))*cos(x)^2 + 3*a^3*atanh(cos(x))*c
os(x)^4 - 7*a*b^2*cos(x)^3 - 10*a^2*b*cos(x)^3 + 15*a*b^2*atanh(cos(x)) + 1
0*a^2*b*atanh(cos(x)) - 30*a*b^2*atanh(cos(x))*cos(x)^2 - 20*a^2*b*atanh(co
s(x))*cos(x)^2 + 15*a*b^2*atanh(cos(x))*cos(x)^4 + 10*a^2*b*atanh(cos(x))*c
os(x)^4)/(8*a^4*cos(x)^4 - 16*a^4*cos(x)^2 + 8*a*b^3 + 24*a^3*b + 8*a^4 + 2
4*a^2*b^2 - 48*a^2*b^2*cos(x)^2 + 24*a^2*b^2*cos(x)^4 - 16*a*b^3*cos(x)^2 -
 48*a^3*b*cos(x)^2 + 8*a*b^3*cos(x)^4 + 24*a^3*b*cos(x)^4)
```

3.17 $\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=88

$$-\frac{(8a^2+20ab+15b^2)\,x}{8b^3}-\frac{(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}+\frac{(4a+7b)\cos(x)\sin(x)}{8b^2}+\frac{\cos(x)\sin^3(x)}{4b}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*x/b^3+1/8*(4*a+7*b)*cos(x)*sin(x)/b^2+1/4*cos(x)
*sin(x)^3/b-(a+b)^(5/2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/b^3/a^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3270, 425, 541, 536, 209, 211}

$$-\frac{x(8a^2+20ab+15b^2)}{8b^3}-\frac{(a+b)^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}+\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}+\frac{\sin^3(x)\cos(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] -1/8*((8*a^2 + 20*a*b + 15*b^2)*x)/b^3 - ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]
*Cot[x])/Sqrt[a]])/(Sqrt[a]*b^3) + ((4*a + 7*b)*Cos[x]*Sin[x])/(8*b^2) + (C
os[x]*Sin[x]^3)/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^6(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(1+x^2)^3\,(a+(a+b)x^2)}\,dx,x,\cot(x)\right)\\ &= \frac{\cos(x)\sin^3(x)}{4b}-\frac{\text{Subst}\left(\int \frac{a+4b-3(a+b)x^2}{(1+x^2)^2(a+(a+b)x^2)}\,dx,x,\cot(x)\right)}{4b}\\ &= \frac{(4a+7b)\cos(x)\sin(x)}{8b^2}+\frac{\cos(x)\sin^3(x)}{4b}-\frac{\text{Subst}\left(\int \frac{4a^2+9ab+8b^2-(a+b)(4a+7b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx,x,\text{co}\right.}{8b^2}\\ &= \frac{(4a+7b)\cos(x)\sin(x)}{8b^2}+\frac{\cos(x)\sin^3(x)}{4b}-\frac{(a+b)^3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{b^3}-\\ &= -\frac{(8a^2+20ab+15b^2)\,x}{8b^3}-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}+\frac{(4a+7b)\cos(x)\sin(x}{8b^2}\end{aligned}$$

Mathematica [A]

time = 0.21, size = 77, normalized size = 0.88

$$\frac{-4(8a^2+20ab+15b^2)\,x+\frac{32(a+b)^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}+8b(a+2b)\sin(2x)-b^2\sin(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (-4*(8*a^2 + 20*a*b + 15*b^2)*x + (32*(a + b)^(5/2)*ArcTan[(Sqrt[a]*Tan[x])
/Sqrt[a + b]])/Sqrt[a] + 8*b*(a + 2*b)*Sin[2*x] - b^2*Sin[4*x])/(32*b^3)
```

Maple [A]
time = 0.18, size = 94, normalized size = 1.07

method	result
default	$-\frac{\frac{\left(-\frac{1}{2}ab-\frac{9}{8}b^2\right)\left(\tan^3(x)\right)+\left(-\frac{1}{2}ab-\frac{7}{8}b^2\right)\tan(x)}{\left(1+\tan^2(x)\right)^2}+\frac{\left(8a^2+20ab+15b^2\right)\arctan(\tan(x))}{8}}{b^3}+\frac{(a+b)^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{b^3\sqrt{(a+b)\,a}}$
risch	$-\frac{x\,a^2}{b^3}-\frac{5ax}{2b^2}-\frac{15x}{8b}-\frac{i\mathrm{e}^{2ix}a}{8b^2}-\frac{i\mathrm{e}^{2ix}}{4b}+\frac{i\mathrm{e}^{-2ix}a}{8b^2}+\frac{i\mathrm{e}^{-2ix}}{4b}-\frac{a\sqrt{-(a+b)\,a}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-(a+b)\,a}+2a+b}{b}\right)}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(((-1/2*a*b-9/8*b^2)*tan(x)^3+(-1/2*a*b-7/8*b^2)*tan(x))/(tan(x)^2+1
)^2+1/8*(8*a^2+20*a*b+15*b^2)*arctan(tan(x)))+(a+b)^3/b^3/((a+b)*a)^(1/2)*a
rctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 112, normalized size = 1.27

$$\frac{(4\,a+9\,b)\tan\left(x\right)^3+(4\,a+7\,b)\tan\left(x\right)}{8\left(b^2\tan\left(x\right)^4+2\,b^2\tan\left(x\right)^2+b^2\right)}-\frac{(8\,a^2+20\,ab+15\,b^2)x}{8\,b^3}+\frac{(a^3+3\,a^2b+3\,ab^2+b^3)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((4*a + 9*b)*tan(x)^3 + (4*a + 7*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(x
)^2 + b^2) - 1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3)
```

Fricas [A]
time = 0.46, size = 285, normalized size = 3.24

$$\left[\frac{2\,{\left(a^{2}+2\,a b+b^{2}\right)} \sqrt{-\frac{a+b}{a}} \log\left(\frac{{\left(8\,a^{2}+8\,a b+b^{2}\right)} \cos\left(x\right)^{4}-2\,{\left(4\,a^{2}+3\,a b\right)} \cos\left(x\right)^{2}-4\,{\left({\left(2\,a^{2}+a b\right)} \cos\left(x\right)^{3}-a^{2} \cos\left(x\right)\right)} \sqrt{-\frac{a+b}{a}} \sin\left(x\right)+a^{2}}{b^{2} \cos\left(x\right)^{4}+2\,a b \cos\left(x\right)^{2}+a^{2}}\right)-{\left(8\,a^{2}+20\,a b+15\,b^{2}\right)} x-{\left(2\,b^{2} \cos\left(x\right)^{3}-{\left(4\,a b+9\,b^{2}\right)} \cos\left(x\right)\right)} \sin\left(x\right)}{8\,b^{3}}, -\frac{4\,{\left(a^{2}+2\,a b+b^{2}\right)} \sqrt{\frac{a+b}{a}} \arctan\left(\frac{{\left({\left(2\,a+b\right)} \cos\left(x\right)^{2}-a\right)} \sqrt{\frac{a+b}{a}}}{2\,{\left(a+b\right)} \cos\left(x\right) \sin\left(x\right)}\right)+{\left(8\,a^{2}+20\,a b+15\,b^{2}\right)} x+{\left(2\,b^{2} \cos\left(x\right)^{3}-{\left(4\,a b+9\,b^{2}\right)} \cos\left(x\right)\right)} \sin\left(x\right)}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*cos(x)
)*sqrt(-(a + b)/a)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - (
8*a^2 + 20*a*b + 15*b^2)*x - (2*b^2*cos(x)^3 - (4*a*b + 9*b^2)*cos(x))*sin(
x))/b^3, -1/8*(4*(a^2 + 2*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*
cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*a*b +
15*b^2)*x + (2*b^2*cos(x)^3 - (4*a*b + 9*b^2)*cos(x))*sin(x))/b^3]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 119, normalized size = 1.35

$$-\frac{{\left(8\,a^{2}+20\,a b+15\,b^{2}\right)} x}{8\,b^{3}}+\frac{{\left(a^{3}+3\,a^{2} b+3\,a b^{2}+b^{3}\right)} {\left(\pi \left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}\left(a\right)+\arctan\left(\frac{a \tan\left(x\right)}{\sqrt{a^{2}+a b}}\right)\right)}}{\sqrt{a^{2}+a b}\, b^{3}}+\frac{4\,a \tan\left(x\right)^{3}+9\,b \tan\left(x\right)^{3}+4\,a \tan\left(x\right)+7\,b \tan\left(x\right)}{8\,{\left(\tan\left(x\right)^{2}+1\right)}^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*
floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*
b)*b^3) + 1/8*(4*a*tan(x)^3 + 9*b*tan(x)^3 + 4*a*tan(x) + 7*b*tan(x))/((tan
(x)^2 + 1)^2*b^2)
```

Mupad [B]
time = 2.68, size = 681, normalized size = 7.74

$$\frac{\frac{{\mathrm{tan}\left(x\right)}^3\,\left(4\,a+9\,b\right)}{8\,b^2}+\frac{\mathrm{tan}\left(x\right)\,\left(4\,a+7\,b\right)}{8\,b^2}}{{\mathrm{tan}\left(x\right)}^4+2\,{\mathrm{tan}\left(x\right)}^2+1}-\frac{\mathrm{atanh}\left(\frac{95\,a^2\,\mathrm{tan}\left(x\right)\,\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3\,b^3-5\,a^2\,b^4-a\,b^5}}{32\,\left(2\,a\,b^4+\frac{469\,a^4\,b}{32}+\frac{215\,a^5}{32}+\frac{287\,a^2\,b^3}{32}+\frac{517\,a^3\,b^2}{32}+\frac{5\,a^6}{4\,b}\right)}+\frac{5\,a^3\,\mathrm{tan}\left(x\right)\,\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3\,b^3-5\,a^2\,b^4-a\,b^5}}{4\,\left(\frac{5\,a^6}{4}+\frac{215\,a^5\,b}{32}+\frac{469\,a^4\,b^2}{32}+\frac{517\,a^3\,b^3}{32}+\frac{287\,a^2\,b^4}{32}+2\,a\,b^5\right)}+\frac{2\,a\,\mathrm{tan}\left(x\right)\,\sqrt{-a^6-5\,a^5\,b-10\,a^4\,b^2-10\,a^3\,b^3-5\,a^2\,b^4-a\,b^5}}{2\,a\,b^3+\frac{517\,a^3\,b}{32}+\frac{469\,a^4}{32}+\frac{287\,a^2\,b^2}{32}+\frac{215\,a^5}{32\,b}+\frac{5\,a^6}{4\,b^2}}\right)\,\sqrt{-a\,{\left(a+b\right)}^5}}{a\,b^3}+\frac{\mathrm{atan}\left(\frac{5717\,a^3\,\mathrm{tan}\left(x\right)}{256\,\left(\frac{15\,a\,b^2}{4}+\frac{3665\,a^2\,b}{256}+\frac{5717\,a^3}{256}+\frac{1143\,a^4}{64\,b}+\frac{235\,a^5}{32\,b^2}+\frac{5\,a^6}{4\,b^3}\right)}+\frac{3665\,a^2\,\mathrm{tan}\left(x\right)}{256\,\left(\frac{15\,a\,b}{4}+\frac{3665\,a^2}{256}+\frac{5717\,a^3}{256\,b}+\frac{1143\,a^4}{64\,b^2}+\frac{235\,a^5}{32\,b^3}+\frac{5\,a^6}{4\,b^4}\right)}+\frac{1143\,a^4\,\mathrm{tan}\left(x\right)}{64\,\left(\frac{15\,a\,b^3}{4}+\frac{5717\,a^3\,b}{256}+\frac{1143\,a^4}{64}+\frac{3665\,a^2\,b^2}{256}+\frac{235\,a^5}{32\,b}+\frac{5\,a^6}{4\,b^2}\right)}+\frac{235\,a^5\,\mathrm{tan}\left(x\right)}{32\,\left(\frac{15\,a\,b^4}{4}+\frac{1143\,a^4\,b}{64}+\frac{235\,a^5}{32}+\frac{3665\,a^2\,b^3}{256}+\frac{5717\,a^3\,b^2}{256}+\frac{5\,a^6}{4\,b}\right)}+\frac{5\,a^6\,\mathrm{tan}\left(x\right)}{4\,\left(\frac{5\,a^6}{4}+\frac{235\,a^5\,b}{32}+\frac{1143\,a^4\,b^2}{64}+\frac{5717\,a^3\,b^3}{256}+\frac{3665\,a^2\,b^4}{256}+\frac{15\,a\,b^5}{4}\right)}+\frac{15\,a\,b\,\mathrm{tan}\left(x\right)}{4\,\left(\frac{15\,a\,b}{4}+\frac{3665\,a^2}{256}+\frac{5717\,a^3}{256\,b}+\frac{1143\,a^4}{64\,b^2}+\frac{235\,a^5}{32\,b^3}+\frac{5\,a^6}{4\,b^4}\right)}\right)\,\left(a^2\,8{}\mathrm{i}+a\,b\,20{}\mathrm{i}+b^2\,15{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6/(a + b*cos(x)^2),x)
```

```
[Out] ((tan(x)^3*(4*a + 9*b))/(8*b^2) + (tan(x)*(4*a + 7*b))/(8*b^2))/(2*tan(x)^2
 + tan(x)^4 + 1) + (atan((5717*a^3*tan(x))/(256*((15*a*b^2)/4 + (3665*a^2*b
)/256 + (5717*a^3)/256 + (1143*a^4)/(64*b) + (235*a^5)/(32*b^2) + (5*a^6)/(
4*b^3))) + (3665*a^2*tan(x))/(256*((15*a*b)/4 + (3665*a^2)/256 + (5717*a^3)
/(256*b) + (1143*a^4)/(64*b^2) + (235*a^5)/(32*b^3) + (5*a^6)/(4*b^4))) + (
1143*a^4*tan(x))/(64*((15*a*b^3)/4 + (5717*a^3*b)/256 + (1143*a^4)/64 + (36
65*a^2*b^2)/256 + (235*a^5)/(32*b) + (5*a^6)/(4*b^2))) + (235*a^5*tan(x))/(
32*((15*a*b^4)/4 + (1143*a^4*b)/64 + (235*a^5)/32 + (3665*a^2*b^3)/256 + (5
717*a^3*b^2)/256 + (5*a^6)/(4*b))) + (5*a^6*tan(x))/(4*((15*a*b^5)/4 + (235
*a^5*b)/32 + (5*a^6)/4 + (3665*a^2*b^4)/256 + (5717*a^3*b^3)/256 + (1143*a^
4*b^2)/64)) + (15*a*b*tan(x))/(4*((15*a*b)/4 + (3665*a^2)/256 + (5717*a^3)/
(256*b) + (1143*a^4)/(64*b^2) + (235*a^5)/(32*b^3) + (5*a^6)/(4*b^4))))*(a*
b*20i + a^2*8i + b^2*15i)*1i)/(8*b^3) - (atanh((95*a^2*tan(x)*(- a*b^5 - 5*
a^5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/(32*(2*a*b^4 + (4
69*a^4*b)/32 + (215*a^5)/32 + (287*a^2*b^3)/32 + (517*a^3*b^2)/32 + (5*a^6)
/(4*b))) + (5*a^3*tan(x)*(- a*b^5 - 5*a^5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3
- 10*a^4*b^2)^(1/2))/(4*(2*a*b^5 + (215*a^5*b)/32 + (5*a^6)/4 + (287*a^2*b^
4)/32 + (517*a^3*b^3)/32 + (469*a^4*b^2)/32)) + (2*a*tan(x)*(- a*b^5 - 5*a^
5*b - a^6 - 5*a^2*b^4 - 10*a^3*b^3 - 10*a^4*b^2)^(1/2))/(2*a*b^3 + (517*a^3
*b)/32 + (469*a^4)/32 + (287*a^2*b^2)/32 + (215*a^5)/(32*b) + (5*a^6)/(4*b^
2)))*(-a*(a + b)^5)^(1/2))/(a*b^3)
```

3.18 $\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{(2a+3b)x}{2b^2}-\frac{(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}+\frac{\cos(x)\sin(x)}{2b}$$

```
[Out] -1/2*(2*a+3*b)*x/b^2+1/2*cos(x)*sin(x)/b-(a+b)^(3/2)*arctan(cot(x)*(a+b)^(1
/2)/a^(1/2))/b^2/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3270, 425, 536, 209, 211}

$$-\frac{(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}-\frac{x(2a+3b)}{2b^2}+\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -1/2*((2*a + 3*b)*x)/b^2 - ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[
a]])/(Sqrt[a]*b^2) + (Cos[x]*Sin[x])/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx,x,\cot(x)\right)\\ &= \frac{\cos(x)\sin(x)}{2b}-\frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx,x,\cot(x)\right)}{2b}\\ &= \frac{\cos(x)\sin(x)}{2b}-\frac{(a+b)^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{b^2}+\frac{(2a+3b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{2b^2}\\ &= -\frac{(2a+3b)x}{2b^2}-\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}+\frac{\cos(x)\sin(x)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 0.87

$$\frac{-4ax-6bx+\frac{4(a+b)^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}+b\sin(2x)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (-4*a*x - 6*b*x + (4*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sq
rt[a] + b*Sin[2*x])/(4*b^2)
```

Maple [A]
time = 0.14, size = 61, normalized size = 1.02

method	result
default	$-\frac{-\frac{b\tan(x)}{2\left(1+\tan^2(x)\right)}+\frac{(2a+3b)\arctan(\tan(x))}{2}}{b^2}+\frac{(a+b)^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{b^2\sqrt{(a+b)\,a}}$
risch	$-\frac{ax}{b^2}-\frac{3x}{2b}-\frac{ie^{2ix}}{8b}+\frac{ie^{-2ix}}{8b}+\frac{\sqrt{-(a+b)\,a}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)\,a}-2a-b}{b}\right)}{2b^2}+\frac{\sqrt{-(a+b)\,a}\,\ln\left(e^{2ix}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/2*b*tan(x)/(tan(x)^2+1)+1/2*(2*a+3*b)*arctan(tan(x)))+(a+b)^2/b^
2/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 62, normalized size = 1.03

$$-\frac{(2\,a+3\,b)x}{2\,b^2}+\frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}+\frac{(a^2+2\,ab+b^2)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + 1/2*tan(x)/(b*tan(x)^2 + b) + (a^2 + 2*a*b + b^2)*
arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2)
```

Fricas [A]
time = 0.45, size = 211, normalized size = 3.52

$$\left[\frac{2\,b\cos(x)\sin(x)+(a+b)\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2-4\left((2\,a^2+ab)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)-2\,(2\,a+3\,b)x}{4\,b^2},\ \frac{b\cos(x)\sin(x)-(a+b)\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a\right)\sqrt{\frac{a+b}{a}}}{2\,(a+b)\cos(x)\sin(x)}\right)-(2\,a+3\,b)x}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*cos(x)*sin(x) + (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^
2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*
cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2
)) - 2*(2*a + 3*b)*x)/b^2, 1/2*(b*cos(x)*sin(x) - (a + b)*sqrt((a + b)/a)*a
```

```
rctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x)))
 - (2*a + 3*b)*x)/b^2]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 80, normalized size = 1.33

$$-\frac{(2a+3b)x}{2b^2}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a^2+2ab+b^2)}{\sqrt{a^2+ab}\,b^2}+\frac{\tan(x)}{2\left(\tan(x)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqr
t(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*b^2) + 1/2*tan(x)/((tan
(x)^2 + 1)*b)
```

Mupad [B]
time = 2.45, size = 126, normalized size = 2.10

$$\frac{\cos(x)\sin(x)}{2b}-\frac{a\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{b^2}-\frac{3\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{2b}-\frac{\mathrm{atanh}\left(\frac{\sin(x)\sqrt{-a^4-3a^3b-3a^2b^2-ab^3}}{\cos(x)\,a^2+2\cos(x)\,a\,b+\cos(x)\,b^2}\right)\sqrt{-a^4-3a^3b-3a^2b^2-ab^3}}{a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4/(a + b*cos(x)^2),x)
```

```
[Out] (cos(x)*sin(x))/(2*b) - (a*atan(sin(x)/cos(x)))/b^2 - (3*atan(sin(x)/cos(x)
))/(2*b) - (atanh((sin(x)*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a^2
*cos(x) + b^2*cos(x) + 2*a*b*cos(x)))*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)
^(1/2))/(a*b^2)
```

3.19 $\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{x}{b}-\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}$$

```
[Out] -x/b-arctan(cot(x)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3270, 400, 209, 211}

$$-\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] -(x/b) - (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{(1+x^2)\,(a+(a+b)x^2)}\,dx, x, \cot(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(x)\right)}{b} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b}\\ &= -\frac{x}{b} - \frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 0.92

$$\frac{-x+\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Cos[x]^2),x]
[Out] (-x + (Sqrt[a + b]*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a])/b
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.90

method	result
default	$-\frac{\arctan(\tan(x))}{b}+\frac{(a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{b\sqrt{(a+b)\,a}}$
risch	$-\frac{x}{b}-\frac{\sqrt{-(a+b)\,a}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-(a+b)\,a}+2a+b}{b}\right)}{2ab}+\frac{\sqrt{-(a+b)\,a}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)\,a}-2a-b}{b}\right)}{2ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/b*arctan(tan(x))+(a+b)/b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2)
)
```

Maxima [A]
time = 0.48, size = 33, normalized size = 0.82

$$\frac{(a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] (a + b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - x/b
```

Fricas [A]
time = 0.45, size = 177, normalized size = 4.42

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2-4\left((2a^2+ab)\cos(x)^3-a^2\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-4x}{4b},-\frac{\sqrt{\frac{a+b}{a}}\arctan\left(\frac{\left((2a+b)\cos(x)^2-a\right)\sqrt{\frac{a+b}{a}}}{2(a+b)\cos(x)\sin(x)}\right)+2x}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - a^2*cos(x))*sqrt(-(a + b)/a)*sin
(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*x)/b, -1/2*(sqrt((a +
 b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt((a + b)/a)/((a + b)*cos(x)*
sin(x))) + 2*x)/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 50, normalized size = 1.25

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(a + b)/(s
qrt(a^2 + a*b)*b) - x/b
```

Mupad [B]
time = 2.37, size = 108, normalized size = 2.70

$$-\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\,\tan(x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\,\tan(x)}{2\,a^2\,b+2\,a\,b^2}\right)}{b}-\frac{\operatorname{atanh}\left(\frac{2\,a^2\,b\,\tan(x)\,\sqrt{-a^2-b\,a}}{2\,a^3\,b+2\,a^2\,b^2}\right)\,\sqrt{-a\,(a+b)}}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*cos(x)^2),x)
```

```
[Out] - atan((2*a*b^2*tan(x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(x))/(2*a*b^2 + 2
*a^2*b))/b - (atanh((2*a^2*b*tan(x)*(- a*b - a^2)^(1/2))/(2*a^3*b + 2*a^2*b
^2))*(-a*(a + b))^(1/2))/(a*b)
```

3.20 $\int \frac{1}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

```
[Out] -arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3260, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-1),x]
```

```
[Out] -(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 29, normalized size = 0.97

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-1),x]
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.70

method	result	[illegible]
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{\sqrt{(a+b)\,a}}$	[illegible]
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}+\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 20, normalized size = 0.67

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
[Out] arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.
time = 0.44, size = 163, normalized size = 5.43

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)}{4(a^2+ab)},\ -\frac{\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2*((
2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(29) = 58$.
time = 19.41, size = 10924, normalized size = 364.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-2*tan(x/
2)/(b*(tan(x/2)**2 - 1)), Eq(a, 0)), (-tan(x/2)/(2*b) + 1/(2*b*tan(x/2)), E
q(a, -b)), (a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-s
qrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*sqrt
(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
 + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-
a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a**3*
sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a + b) + b/
(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/
(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqr
t(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqrt(-a*b)*sqrt
(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
 + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)
/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2
))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a
+ b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b
))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(
-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*
b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*
sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) + a**3*sqrt(-a/(a + b) + b/(a + b)
 + 2*sqrt(-a*b)/(a + b))*log(-sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a
 + b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a
```

```
/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) +
2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*s
qrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10
*a**2*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a +
 b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a +
 b)) + 8*a*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a
+ b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) -
2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*s
qrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10
*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
 + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) +
b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b
)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b
))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(
a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*
sqrt(-a*b)/(a + b)) + 8*a*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*s
qrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - 1
0*a**2*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-sqrt(-a/(
a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*sqrt(-a/(a +
 b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqr
t(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a
+ b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*
b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/
(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))
 + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a
+ b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqrt(-a*b)*sqrt(-a/(a +
 b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqr
t(-a*b)/(a + b))) + 10*a**2*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a
 + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/
(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
 + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b
) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b
)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*
sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/
(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2
*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b)...
```

Giac [A]
time = 0.42, size = 37, normalized size = 1.23

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2 +
 a*b)
```

Mupad [B]
time = 2.38, size = 24, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2),x)
```

```
[Out] atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

3.21 $\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=41

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}-\frac{\cot(x)}{a+b}$$

```
[Out] -cot(x)/(a+b)-b*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/(a+b)^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3270, 396, 211}

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}-\frac{\cot(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2))) - Cot[x
]/(a + b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx, x, \cot(x)\right)$$
$$= -\frac{\cot(x)}{a+b} - \frac{b\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{a+b}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}} - \frac{\cot(x)}{a+b}$$

Mathematica [A]
time = 0.11, size = 40, normalized size = 0.98

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}} - \frac{\cot(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(3/2)) - Cot[x]/(
a + b)
```

Maple [A]
time = 0.15, size = 39, normalized size = 0.95

method	result
default	$-\frac{1}{(a+b)\tan(x)} + \frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{(a+b)\sqrt{(a+b)\,a}}$
risch	$-\frac{2i}{(e^{2ix}-1)(a+b)} + \frac{b\ln\left(e^{2ix}+\frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)} - \frac{b\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}\cdots}{b\sqrt{-a^2-\cdots}}\cdots\right)}{2\sqrt{-a^2-ab}\,(a+\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a+b)/tan(x)+b/(a+b)/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 38, normalized size = 0.93

$$\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)} - \frac{1}{(a+b)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)) - 1/((a + b)*t
an(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(33) = 66$.
time = 0.45, size = 228, normalized size = 5.56

$$\left[-\frac{\sqrt{-a^2-ab}\, b \log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2+4\left((2a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)\sin(x)+4(a^2+ab)\cos(x)}{4(a^3+2a^2b+ab^2)\sin(x)}, -\frac{\sqrt{a^2+ab}\, b \arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)\sin(x)+2(a^2+ab)\cos(x)}{2(a^3+2a^2b+ab^2)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 +
3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x)
 + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) + 4*(a^2 + a*b)*cos(x
))/((a^3 + 2*a^2*b + a*b^2)*sin(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2
*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*sin(x) + 2*(a^2 + a*
b)*cos(x))/((a^3 + 2*a^2*b + a*b^2)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a+b\cos^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.44, size = 55, normalized size = 1.34

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}\,(a+b)}-\frac{1}{(a+b)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b/(sqrt(a^
2 + a*b)*(a + b)) - 1/((a + b)*tan(x))
```

Mupad [B]
time = 2.30, size = 34, normalized size = 0.83

$$\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,\left(a+b\right)^{3/2}}-\frac{1}{\tan\left(x\right)\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*cos(x)^2)),x)
```

```
[Out] (b*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(1/2)*(a + b)^(3/2)) - 1/(tan(x
)*(a + b))
```

3.22 $\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{(a+2b)\cot(x)}{(a+b)^2}-\frac{\cot^3(x)}{3(a+b)}$$

```
[Out] -(a+2*b)*cot(x)/(a+b)^2-1/3*cot(x)^3/(a+b)-b^2*arctan(cot(x)*(a+b)^(1/2)/a^
(1/2))/(a+b)^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3270, 398, 211}

$$-\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{\cot^3(x)}{3(a+b)}-\frac{(a+2b)\cot(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^2*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2))) - ((a
 + 2*b)*Cot[x])/(a + b)^2 - Cot[x]^3/(3*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2}\,dx,x,\cot(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2\,(a+(a+b)x^2)}\right)dx,x,\cot(x)\right)\\
&= -\frac{(a+2b)\cot(x)}{(a+b)^2}-\frac{\cot^3(x)}{3(a+b)}-\frac{b^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{(a+b)^2}\\
&= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{(a+2b)\cot(x)}{(a+b)^2}-\frac{\cot^3(x)}{3(a+b)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.24, size = 59, normalized size = 0.97

$$
\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{5/2}}-\frac{\cot(x)\,(2a+5b+(a+b)\csc^2(x))}{3(a+b)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) - (Cot[x
]*(2*a + 5*b + (a + b)*Csc[x]^2))/(3*(a + b)^2)
```

Maple [**A**]
time = 0.18, size = 57, normalized size = 0.93

method	result
default	$-\frac{1}{3(a+b)\tan(x)^3}-\frac{a+2b}{(a+b)^2\tan(x)}+\frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{(a+b)^2\sqrt{(a+b)\,a}}$
risch	$-\frac{2i(3b\,\mathrm{e}^{4ix}-6a\,\mathrm{e}^{2ix}-12b\,\mathrm{e}^{2ix}+2a+5b)}{3(\mathrm{e}^{2ix}-1)^3(a+b)^2}-\frac{b^2\ln\left(\mathrm{e}^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)^2}+\frac{b^2\ln\left(\mathrm{e}^{2ix}+\frac{-2ia^2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a+b)/tan(x)^3-(a+2*b)/(a+b)^2/tan(x)+b^2/(a+b)^2/((a+b)*a)^(1/2)*arct
an(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 70, normalized size = 1.15

$$\frac{b^2 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)} - \frac{3\,(a+2\,b)\tan\left(x\right)^2+a+b}{3\,(a^2+2\,ab+b^2)\tan\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2))
- 1/3*(3*(a + 2*b)*tan(x)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(x)^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(51) = 102.
time = 0.46, size = 396, normalized size = 6.49

$$\left[\frac{4\,(2\,a^3+7\,a^2b+5\,ab^2)\cos\left(x\right)^3+3\,(b^2\cos\left(x\right)^2-b^2)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2+4\,\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)\sin\left(x\right)-12\,(a^3+3\,a^2b+2\,ab^2)\cos\left(x\right)}{12\,\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^4+3\,a^3b+3\,a^2b^2+ab^3)\cos\left(x\right)^2\right)\sin\left(x\right)}, \frac{2\,(2\,a^3+7\,a^2b+5\,ab^2)\cos\left(x\right)^3+3\,(b^2\cos\left(x\right)^2-b^2)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\,\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)\sin\left(x\right)-6\,(a^3+3\,a^2b+2\,ab^2)\cos\left(x\right)}{6\,\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-(a^4+3\,a^3b+3\,a^2b^2+ab^3)\cos\left(x\right)^2\right)\sin\left(x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^3 + 3*(b^2*cos(x)^2 - b^2)*sqrt
(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)
^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b^2*
cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) - 12*(a^3 + 3*a^2*b + 2*a*b^2)*cos
(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*cos(x)^2)*sin(x)), 1/6*(2*(2*a^3 + 7*a^2*b + 5*a*b^2)*cos(x)^3 + 3*(b^2
*cos(x)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(
a^2 + a*b)*cos(x)*sin(x)))*sin(x) - 6*(a^3 + 3*a^2*b + 2*a*b^2)*cos(x))/((a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(
x)^2)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(x\right)}{a+b\cos^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**4/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.43, size = 90, normalized size = 1.48

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{(a^2+2\,ab+b^2)\sqrt{a^2+ab}}-\frac{3\,a\tan\left(x\right)^2+6\,b\tan\left(x\right)^2+a+b}{3\,(a^2+2\,ab+b^2)\tan\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^2/((a^2
+ 2*a*b + b^2)*sqrt(a^2 + a*b)) - 1/3*(3*a*tan(x)^2 + 6*b*tan(x)^2 + a + b)
/((a^2 + 2*a*b + b^2)*tan(x)^3)
```

Mupad [B]
time = 2.34, size = 67, normalized size = 1.10

$$\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)\,(a^2+2\,a\,b+b^2)}{(a+b)^{5/2}}\right)}{\sqrt{a}\,\left(a+b\right)^{5/2}}-\frac{\frac{1}{3\,(a+b)}+\frac{\tan(x)^2\,(a+2\,b)}{(a+b)^2}}{\tan\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4*(a + b*cos(x)^2)),x)
```

```
[Out] (b^2*atan((a^(1/2)*tan(x)*(2*a*b + a^2 + b^2))/(a + b)^(5/2)))/(a^(1/2)*(a
+ b)^(5/2)) - (1/(3*(a + b)) + (tan(x)^2*(a + 2*b))/(a + b)^2)/tan(x)^3
```

3.23 $\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=89

$$-\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{(a^2+3ab+3b^2)\cot(x)}{(a+b)^3}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}-\frac{\cot^5(x)}{5(a+b)}$$

```
[Out] -(a^2+3*a*b+3*b^2)*cot(x)/(a+b)^3-1/3*(2*a+3*b)*cot(x)^3/(a+b)^2-1/5*cot(x)
^5/(a+b)-b^3*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/(a+b)^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3270, 398, 211}

$$-\frac{(a^2+3ab+3b^2)\cot(x)}{(a+b)^3}-\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{\cot^5(x)}{5(a+b)}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^3*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2))) - ((a
^2 + 3*a*b + 3*b^2)*Cot[x])/(a + b)^3 - ((2*a + 3*b)*Cot[x]^3)/(3*(a + b)^2
) - Cot[x]^5/(5*(a + b))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{(1+x^2)^3}{a+(a+b)x^2}\,dx,x,\cot(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3}+\frac{(2a+3b)x^2}{(a+b)^2}+\frac{x^4}{a+b}+\frac{b^3}{(a+b)^3\,(a+(a+b)x^2)}\right)dx,$$
$$= -\frac{(a^2+3ab+3b^2)\cot(x)}{(a+b)^3}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}-\frac{\cot^5(x)}{5(a+b)}-\frac{b^3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,\right.}{(a+b)^3}$$
$$= -\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{(a^2+3ab+3b^2)\cot(x)}{(a+b)^3}-\frac{(2a+3b)\cot^3(x)}{3(a+b)^2}-\frac{\cot^5(x)}{5(a+b}$$

Mathematica [A]
time = 0.43, size = 90, normalized size = 1.01

$$\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{7/2}}-\frac{\cot(x)\,(8a^2+26ab+33b^2+(4a^2+13ab+9b^2)\csc^2(x)+3(a+b)^2\csc^4(x))}{15(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(7/2)) - (Cot[x
]*(8*a^2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Csc[x]^2 + 3*(a + b)^
2*Csc[x]^4))/(15*(a + b)^3)
```

Maple [A]
time = 0.20, size = 83, normalized size = 0.93

method	result
default	$-\frac{1}{5(a+b)\tan(x)^5}-\frac{2a+3b}{3(a+b)^2\tan(x)^3}-\frac{a^2+3ab+3b^2}{(a+b)^3\tan(x)}+\frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{(a+b)^3\sqrt{(a+b)\,a}}$
risch	$-\frac{2i(15b^2\mathrm{e}^{8ix}-30ab\,\mathrm{e}^{6ix}-90b^2\mathrm{e}^{6ix}+80a^2\mathrm{e}^{4ix}+230ab\,\mathrm{e}^{4ix}+240b^2\mathrm{e}^{4ix}-40a^2\mathrm{e}^{2ix}-130b\,\mathrm{e}^{2ix}a-150b^2\mathrm{e}^{2ix}+8a^2+26ab+33b^2)}{15(a+b)^3(\mathrm{e}^{2ix}-1)^5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(a+b)/tan(x)^5-1/3*(2*a+3*b)/(a+b)^2/tan(x)^3-(a^2+3*a*b+3*b^2)/(a+b)^
3/tan(x)+b^3/(a+b)^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 127, normalized size = 1.43

$$\frac{b^3 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{(a+b)a}}-\frac{15\,(a^2+3\,ab+3\,b^2)\tan(x)^4+5\,(2\,a^2+5\,ab+3\,b^2)\tan(x)^2+3\,a^2+6\,ab+3\,b^2}{15\,(a^3+3\,a^2b+3\,ab^2+b^3)\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b^3*arctan(a*tan(x)/sqrt((a + b)*a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(
(a + b)*a)) - 1/15*(15*(a^2 + 3*a*b + 3*b^2)*tan(x)^4 + 5*(2*a^2 + 5*a*b +
3*b^2)*tan(x)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*t
an(x)^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(77) = 154.
time = 0.45, size = 610, normalized size = 6.85

$$\left[-\frac{4\,(8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3)\cos(x)^5-20\,(4\,a^4+17\,a^3b+28\,a^2b^2+15\,ab^3)\cos(x)^3+15\,(b^3\cos(x)^4-2\,b^3\cos(x)^2+b^3)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2+4\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)\sin(x)+60\,(a^4+4\,a^3b+6\,a^2b^2+3\,ab^3)\cos(x)}{60\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\cos(x)^4-2\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\cos(x)^2)\sin(x)},\ -\frac{2\,(8\,a^4+34\,a^3b+59\,a^2b^2+33\,ab^3)\cos(x)^5-10\,(4\,a^4+17\,a^3b+28\,a^2b^2+15\,ab^3)\cos(x)^3+15\,(b^3\cos(x)^4-2\,b^3\cos(x)^2+b^3)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\,\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)\sin(x)+30\,(a^4+4\,a^3b+6\,a^2b^2+3\,ab^3)\cos(x)}{30\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\cos(x)^4-2\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)\cos(x)^2)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^5 - 20*(4*a^4 +
 17*a^3*b + 28*a^2*b^2 + 15*a*b^3)*cos(x)^3 + 15*(b^3*cos(x)^4 - 2*b^3*cos(
x)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2
 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin
(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))*sin(x) + 60*(a^4 + 4*a^3*
b + 6*a^2*b^2 + 3*a*b^3)*cos(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^4 - 2*(a^5 +
 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)*sin(x)), -1/30*(2*(8*a^
4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^5 - 10*(4*a^4 + 17*a^3*b + 28*
a^2*b^2 + 15*a*b^3)*cos(x)^3 + 15*(b^3*cos(x)^4 - 2*b^3*cos(x)^2 + b^3)*sqr
t(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*si
n(x)))*sin(x) + 30*(a^4 + 4*a^3*b + 6*a^2*b^2 + 3*a*b^3)*cos(x))/((a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*
b^3 + a*b^4)*cos(x)^4 - 2*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*c
os(x)^2)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Integral(csc(x)**6/(a + b*cos(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(77) = 154.
time = 0.43, size = 156, normalized size = 1.75

$$\frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan(x)}{\sqrt{a^2 + ab}}\right)\right) b^3}{(a^3 + 3\,a^2 b + 3\,a b^2 + b^3)\sqrt{a^2 + ab}} - \frac{15\,a^2 \tan{(x)}^4 + 45\,ab \tan{(x)}^4 + 45\,b^2 \tan{(x)}^4 + 10\,a^2 \tan{(x)}^2 + 25\,ab \tan{(x)}^2 + 15\,b^2 \tan{(x)}^2 + 3\,a^2 + 6\,ab + 3\,b^2}{15\,(a^3 + 3\,a^2 b + 3\,a b^2 + b^3) \tan{(x)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^3/((a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) - 1/15*(15*a^2*tan(x)^4 + 45*a*
b*tan(x)^4 + 45*b^2*tan(x)^4 + 10*a^2*tan(x)^2 + 25*a*b*tan(x)^2 + 15*b^2*t
an(x)^2 + 3*a^2 + 6*a*b + 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(x)^5)
```

Mupad [B]
time = 2.37, size = 101, normalized size = 1.13

$$\frac{b^3 \operatorname{atan}\left(\frac{\sqrt{a}\,\tan(x)\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)}{(a+b)^{7/2}}\right)}{\sqrt{a}\,(a+b)^{7/2}} - \frac{\frac{1}{5\,(a+b)} + \frac{\tan(x)^2\,(2\,a+3\,b)}{3\,(a+b)^2} + \frac{\tan(x)^4\,(a^2+3\,a\,b+3\,b^2)}{(a+b)^3}}{\tan{(x)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^6*(a + b*cos(x)^2)),x)
```

```
[Out] (b^3*atan((a^(1/2)*tan(x)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(a + b)^(7/2)))/
(a^(1/2)*(a + b)^(7/2)) - (1/(5*(a + b)) + (tan(x)^2*(2*a + 3*b))/(3*(a + b
)^2) + (tan(x)^4*(3*a*b + a^2 + 3*b^2))/(a + b)^3)/tan(x)^5
```

3.24 $\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx$

Optimal. Leaf size=98

$$-\frac{\text{ArcTan}\left(\frac{1+\sqrt[3]{6}\cos(x)}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}}+\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}-\frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\cos(x)+3^{2/3}\cos^2(x)\right)}{12\sqrt[3]{6}}$$

```
[Out] -1/12*arctan(1/3*(1+6^(1/3)*cos(x))*3^(1/2))*2^(2/3)*3^(1/6)+1/36*ln(2^(2/3
)-3^(1/3)*cos(x))*6^(2/3)-1/72*ln(2*2^(1/3)+2^(2/3)*3^(1/3)*cos(x)+3^(2/3)*
cos(x)^2)*6^(2/3)
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3302, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{6}\cos(x)+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}}-\frac{\log\left(3^{2/3}\cos^2(x)+2^{2/3}\sqrt[3]{3}\cos(x)+2\sqrt[3]{2}\right)}{12\sqrt[3]{6}}+\frac{\log\left(2^{2/3}-\sqrt[3]{3}\cos(x)\right)}{6\sqrt[3]{6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(4 - 3*Cos[x]^3),x]
```

```
[Out] -1/2*ArcTan[(1 + 6^(1/3)*Cos[x])/Sqrt[3]]/(2^(1/3)*3^(5/6)) + Log[2^(2/3) -
 3^(1/3)*Cos[x]]/(6*6^(1/3)) - Log[2*2^(1/3) + 2^(2/3)*3^(1/3)*Cos[x] + 3^(
2/3)*Cos[x]^2]/(12*6^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sin(x)}{4-3\cos^3(x)}\,dx = -\text{Subst}\left(\int \frac{1}{4-3x^3}\,dx, x, \cos(x)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{3}\,x}\,dx, x, \cos(x)\right)}{6\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{2\,2^{2/3}+\sqrt[3]{3}\,x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \cos(x)\right)}{6\sqrt[3]{2}}$$

$$= \frac{\log\left(2^{2/3}-\sqrt[3]{3}\,\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\,x+3^{2/3}x^2}\,dx, x, \cos(x)\right)}{2\,2^{2/3}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(2^{2/3}-\sqrt[3]{3}\,\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\,\cos(x)+3^{2/3}\cos^2(x)\right)}{12\sqrt[3]{6}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{6}\,\cos(x)}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,3^{5/6}} + \frac{\log\left(2^{2/3}-\sqrt[3]{3}\,\cos(x)\right)}{6\sqrt[3]{6}} - \frac{\log\left(2\sqrt[3]{2}+2^{2/3}\sqrt[3]{3}\,\cos(x)+\right.}{12\sqrt[3]{6}}$$

Mathematica [A]
time = 0.10, size = 79, normalized size = 0.81

$$\frac{1}{72}\left(-62^{2/3}\sqrt[6]{3}\,\text{ArcTan}\left(\frac{1+\sqrt[3]{6}\cos(x)}{\sqrt{3}}\right)+6^{2/3}\left(2\log\left(2-\sqrt[3]{6}\cos(x)\right)-\log\left(4+2\sqrt[3]{6}\cos(x)+6^{2/3}\cos^2(x)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(4 - 3*Cos[x]^3),x]
```

```
[Out] (-6*2^(2/3)*3^(1/6)*ArcTan[(1 + 6^(1/3)*Cos[x])/Sqrt[3]] + 6^(2/3)*(2*Log[2
 - 6^(1/3)*Cos[x]] - Log[4 + 2*6^(1/3)*Cos[x] + 6^(2/3)*Cos[x]^2]))/72
```

Maple [A]
time = 0.13, size = 80, normalized size = 0.82

method	result
risch	$-\frac{i\left(\sum_{\_R=\mathrm{RootOf}\left(162\_Z^3+i\right)}\_R\ln\left(e^{2ix}+12i\_R\,e^{ix}+1\right)\right)}{2}$
derivativedivides	$\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos^2(x)+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cos(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}-\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cos(x)}{2}+1\right)}{3}\right)}{12}$
default	$\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos(x)-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{36}-\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\cos^2(x)+\frac{4^{\frac{1}{3}}3^{\frac{2}{3}}\cos(x)}{3}+\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{72}-\frac{4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{4^{\frac{2}{3}}3^{\frac{1}{3}}\cos(x)}{2}+1\right)}{3}\right)}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(4-3*cos(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*4^(1/3)*3^(2/3)*ln(cos(x)-1/3*4^(1/3)*3^(2/3))-1/72*4^(1/3)*3^(2/3)*ln
(cos(x)^2+1/3*4^(1/3)*3^(2/3)*cos(x)+1/3*4^(2/3)*3^(1/3))-1/12*4^(1/3)*3^(1
/6)*arctan(1/3*3^(1/2)*(1/2*4^(2/3)*3^(1/3)*cos(x)+1))
```

Maxima [A]
time = 0.47, size = 89, normalized size = 0.91

$$-\frac{1}{72}\cdot 4^{\frac{1}{3}}3^{\frac{2}{3}}\log\left(3^{\frac{2}{3}}\cos(x)^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\cos(x)+4^{\frac{2}{3}}\right)+\frac{1}{36}\cdot 4^{\frac{1}{3}}3^{\frac{2}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\cos(x)-4^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 4^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}3^{\frac{1}{6}}\left(2\cdot 3^{\frac{2}{3}}\cos(x)+4^{\frac{1}{3}}3^{\frac{1}{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="maxima")
```

```
[Out] -1/72*4^(1/3)*3^(2/3)*log(3^(2/3)*cos(x)^2 + 4^(1/3)*3^(1/3)*cos(x) + 4^(2/
3)) + 1/36*4^(1/3)*3^(2/3)*log(1/3*3^(2/3)*(3^(1/3)*cos(x) - 4^(1/3))) - 1/
```

```
12*4^(1/3)*3^(1/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*3^(2/3)*cos(x) + 4^(1/3)*
3^(1/3)))
```

Fricas [A]
time = 0.41, size = 71, normalized size = 0.72

$$-\frac{1}{12}\cdot 6^{\frac{1}{6}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 6^{\frac{1}{6}}\left(6^{\frac{2}{3}}\sqrt{2}\cos\left(x\right)+6^{\frac{1}{3}}\sqrt{2}\right)\right)-\frac{1}{72}\cdot 6^{\frac{2}{3}}\log\left(-3\cos\left(x\right)^{2}-6^{\frac{2}{3}}\cos\left(x\right)-2\cdot 6^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 6^{\frac{2}{3}}\log\left(6^{\frac{2}{3}}-3\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*6^(1/6)*sqrt(2)*arctan(1/6*6^(1/6)*(6^(2/3)*sqrt(2)*cos(x) + 6^(1/3)*
sqrt(2))) - 1/72*6^(2/3)*log(-3*cos(x)^2 - 6^(2/3)*cos(x) - 2*6^(1/3)) + 1/
36*6^(2/3)*log(6^(2/3) - 3*cos(x))
```

Sympy [A]
time = 0.54, size = 85, normalized size = 0.87

$$\frac{6^{\frac{2}{3}}\log\left(\cos\left(x\right)-\frac{6^{\frac{2}{3}}}{3}\right)}{36}-\frac{6^{\frac{2}{3}}\log\left(36\cos^{2}\left(x\right)+12\cdot 6^{\frac{2}{3}}\cos\left(x\right)+24\cdot\sqrt[3]{6}\right)}{72}-\frac{2^{\frac{2}{3}}\cdot\sqrt[6]{3}\operatorname{atan}\left(\frac{\sqrt[3]{2}\cdot 3^{\frac{5}{6}}\cos\left(x\right)}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)**3),x)
```

```
[Out] 6**(2/3)*log(cos(x) - 6**(2/3)/3)/36 - 6**(2/3)*log(36*cos(x)**2 + 12*6**(2
/3)*cos(x) + 24*6**(1/3))/72 - 2**(2/3)*3**(1/6)*atan(2**(1/3)*3**(5/6)*cos
(x)/3 + sqrt(3)/3)/12
```

Giac [A]
time = 0.39, size = 60, normalized size = 0.61

$$-\frac{1}{12}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{4}\sqrt{3}\left(\frac{4}{3}\right)^{\frac{2}{3}}\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}+2\cos\left(x\right)\right)\right)-\frac{1}{72}\cdot 36^{\frac{1}{3}}\log\left(\cos\left(x\right)^{2}+\left(\frac{4}{3}\right)^{\frac{1}{3}}\cos\left(x\right)+\left(\frac{4}{3}\right)^{\frac{2}{3}}\right)+\frac{1}{12}\left(\frac{4}{3}\right)^{\frac{1}{3}}\log\left(\left(\frac{4}{3}\right)^{\frac{1}{3}}-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(4-3*cos(x)^3),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*(4/3)^(1/3)*arctan(1/4*sqrt(3)*(4/3)^(2/3)*((4/3)^(1/3) + 2*c
os(x))) - 1/72*36^(1/3)*log(cos(x)^2 + (4/3)^(1/3)*cos(x) + (4/3)^(2/3)) +
1/12*(4/3)^(1/3)*log((4/3)^(1/3) - cos(x))
```

Mupad [B]
time = 0.31, size = 75, normalized size = 0.77

$$\frac{6^{2/3}\ln\left(\cos\left(x\right)-\frac{6^{2/3}}{3}\right)}{36}+\frac{6^{2/3}\ln\left(\cos\left(x\right)-\frac{6^{2/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{72}-\frac{6^{2/3}\ln\left(\cos\left(x\right)+\frac{6^{2/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(3*cos(x)^3 - 4),x)
```

```
[Out] (6^(2/3)*log(cos(x) - 6^(2/3)/3))/36 + (6^(2/3)*log(cos(x) - (6^(2/3)*(3^(1
/2)*1i - 1))/6)*(3^(1/2)*1i - 1))/72 - (6^(2/3)*log(cos(x) + (6^(2/3)*(3^(1
/2)*1i + 1))/6)*(3^(1/2)*1i + 1))/72
```

3.25 $\int \frac{1}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=4

$$-\cot(x)$$

[Out] -cot(x)

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3254, 3852, 8}

$$-\cot(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - Cos[x]^2)^(-1),x]

[Out] -Cot[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cos^2(x)}\,dx &= \int \csc^2(x)\,dx\\ &= -\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)\\ &= -\cot(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-1),x]
[Out] -Cot[x]
```

Maple [A]
time = 0.04, size = 7, normalized size = 1.75

method	result	size
default	$-\frac{1}{\tan(x)}$	7
risch	$-\frac{2i}{e^{2ix}-1}$	13
norman	$\frac{-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/tan(x)
```

Maxima [A]
time = 0.26, size = 6, normalized size = 1.50

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2),x, algorithm="maxima")
[Out] -1/tan(x)
```

Fricas [A]
time = 0.44, size = 8, normalized size = 2.00

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2),x, algorithm="fricas")
[Out] -cos(x)/sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.
time = 0.20, size = 14, normalized size = 3.50

$$\frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2),x)
```

```
[Out] tan(x/2)/2 - 1/(2*tan(x/2))
```

Giac [A]
time = 0.41, size = 6, normalized size = 1.50

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/tan(x)
```

Mupad [B]
time = 2.24, size = 4, normalized size = 1.00

$$-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2 - 1),x)
```

```
[Out] -cot(x)
```

3.26 $\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx$

Optimal. Leaf size=13

$$-\cot(x)-\frac{\cot^3(x)}{3}$$

```
[Out] -cot(x)-1/3*cot(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3254, 3852}

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-2),x]
```

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-\cos^2(x)\right)^2} \, dx &= \int \csc^4(x) \, dx\\ &= -\text{Subst}\left(\int \left(1+x^2\right) \, dx,x,\cot(x)\right)\\ &= -\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.31

$$-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-2),x]
[Out] (-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A]
time = 0.05, size = 14, normalized size = 1.08

method	result	size
default	$-\frac{1}{3\tan(x)^3}-\frac{1}{\tan(x)}$	14
risch	$\frac{4i(3\,e^{2ix}-1)}{3(e^{2ix}-1)^3}$	22
norman	$\frac{-\frac{1}{24}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/3/tan(x)^3-1/tan(x)
```

Maxima [A]
time = 0.27, size = 14, normalized size = 1.08

$$-\frac{3\,\tan\left(x\right)^2+1}{3\,\tan\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^2,x, algorithm="maxima")
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.41, size = 25, normalized size = 1.92

$$-\frac{2\,\cos\left(x\right)^3-3\,\cos\left(x\right)}{3\,\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.
time = 0.48, size = 34, normalized size = 2.62

$$\frac{\tan^3\left(\frac{x}{2}\right)}{24} + \frac{3\tan\left(\frac{x}{2}\right)}{8} - \frac{3}{8\tan\left(\frac{x}{2}\right)} - \frac{1}{24\tan^3\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**2,x)
```

```
[Out] tan(x/2)**3/24 + 3*tan(x/2)/8 - 3/(8*tan(x/2)) - 1/(24*tan(x/2)**3)
```

Giac [A]
time = 0.41, size = 14, normalized size = 1.08

$$-\frac{3\tan\left(x\right)^2 + 1}{3\tan\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

Mupad [B]
time = 2.25, size = 10, normalized size = 0.77

$$-\frac{\cot(x)\,\left(\cot(x)^2 + 3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - 1)^2,x)
```

```
[Out] -(cot(x)*(cot(x)^2 + 3))/3
```

3.27 $\int \frac{1}{\left(1-\cos^2(x)\right)^3} dx$

Optimal. Leaf size=21

$$-\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}$$

```
[Out] -cot(x)-2/3*cot(x)^3-1/5*cot(x)^5
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3254, 3852}

$$-\frac{1}{5}\cot^5(x) - \frac{2\cot^3(x)}{3} - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-3),x]
```

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-\cos^2(x))^3} dx &= \int \csc^6(x)\, dx \\
&= -\text{Subst}\left(\int \left(1+2x^2+x^4\right) dx, x, \cot(x)\right) \\
&= -\cot(x) - \frac{2\cot^3(x)}{3} - \frac{\cot^5(x)}{5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.29

$$-\frac{8\cot(x)}{15}-\frac{4}{15}\cot(x)\csc^2(x)-\frac{1}{5}\cot(x)\csc^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-3),x]
```

```
[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Maple [A]
time = 0.07, size = 20, normalized size = 0.95

method	result	size
default	$-\frac{1}{\tan(x)}-\frac{1}{5\tan(x)^5}-\frac{2}{3\tan(x)^3}$	20
risch	$-\frac{16i(10\,\mathrm{e}^{4ix}-5\,\mathrm{e}^{2ix}+1)}{15(\mathrm{e}^{2ix}-1)^5}$	29
norman	$\frac{-\frac{1}{160}-\frac{5\left(\tan^2\left(\frac{x}{2}\right)\right)}{96}-\frac{5\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^6\left(\frac{x}{2}\right)\right)}{16}+\frac{5\left(\tan^8\left(\frac{x}{2}\right)\right)}{96}+\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{160}}{\tan\left(\frac{x}{2}\right)^5}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/tan(x)-1/5/tan(x)^5-2/3/tan(x)^3
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.95

$$-\frac{15\tan(x)^4+10\tan(x)^2+3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.41, size = 37, normalized size = 1.76

$$-\frac{8\cos(x)^5-20\cos(x)^3+15\cos(x)}{15\left(\cos(x)^4-2\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*s
in(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(19) = 38$.

time = 1.28, size = 54, normalized size = 2.57

$$\frac{\tan^5\left(\frac{x}{2}\right)}{160}+\frac{5\tan^3\left(\frac{x}{2}\right)}{96}+\frac{5\tan\left(\frac{x}{2}\right)}{16}-\frac{5}{16\tan\left(\frac{x}{2}\right)}-\frac{5}{96\tan^3\left(\frac{x}{2}\right)}-\frac{1}{160\tan^5\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**3,x)
```

```
[Out] tan(x/2)**5/160 + 5*tan(x/2)**3/96 + 5*tan(x/2)/16 - 5/(16*tan(x/2)) - 5/(9
6*tan(x/2)**3) - 1/(160*tan(x/2)**5)
```

Giac [A]

time = 0.41, size = 20, normalized size = 0.95

$$-\frac{15\,\tan\left(x\right)^4+10\,\tan\left(x\right)^2+3}{15\,\tan\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

Mupad [B]

time = 2.24, size = 17, normalized size = 0.81

$$-\frac{\cot(x)^5}{5}-\frac{2\cot(x)^3}{3}-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2 - 1)^3,x)
```

```
[Out] - cot(x) - (2*cot(x)^3)/3 - cot(x)^5/5
```

3.28 $\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{(a^2-ab+b^2)\sin(x)}{b^3}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}$$

```
[Out] (a^2-a*b+b^2)*sin(x)/b^3+1/3*(a-2*b)*sin(x)^3/b^2+1/5*sin(x)^5/b-a^3*arctan
h(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3265, 398, 214}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}}+\frac{(a^2-ab+b^2)\sin(x)}{b^3}+\frac{(a-2b)\sin^3(x)}{3b^2}+\frac{\sin^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] -((a^3*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b])) + ((a^
2 - a*b + b^2)*Sin[x])/b^3 + ((a - 2*b)*Sin[x]^3)/(3*b^2) + Sin[x]^5/(5*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^7(x)}{a+b\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-bx^2}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{b^3} + \frac{(a-2b)x^2}{b^2} + \frac{x^4}{b} - \frac{a^3}{b^3(a+b-bx^2)}\right)\,dx, x, \sin(x)\right)$$
$$= \frac{(a^2-ab+b^2)\sin(x)}{b^3} + \frac{(a-2b)\sin^3(x)}{3b^2} + \frac{\sin^5(x)}{5b} - \frac{a^3\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{b^3}$$
$$= -\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}} + \frac{(a^2-ab+b^2)\sin(x)}{b^3} + \frac{(a-2b)\sin^3(x)}{3b^2} + \frac{\sin^5(x)}{5b}$$

Mathematica [**A**]
time = 0.45, size = 111, normalized size = 1.42

$$\frac{a^3\left(\log\left(\sqrt{a+b}-\sqrt{b}\,\sin(x)\right)-\log\left(\sqrt{a+b}+\sqrt{b}\,\sin(x)\right)\right)}{2b^{7/2}\sqrt{a+b}} + \frac{(8a^2-6ab+5b^2)\sin(x)}{8b^3} + \frac{(-4a+5b)\sin(3x)}{48b^2} + \frac{\sin(5x)}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a + b*Cos[x]^2),x]
```

```
[Out] (a^3*(Log[Sqrt[a + b] - Sqrt[b]*Sin[x]] - Log[Sqrt[a + b] + Sqrt[b]*Sin[x]]
))/(2*b^(7/2)*Sqrt[a + b]) + ((8*a^2 - 6*a*b + 5*b^2)*Sin[x])/(8*b^3) + ((-
4*a + 5*b)*Sin[3*x])/(48*b^2) + Sin[5*x]/(80*b)
```

Maple [**A**]
time = 0.24, size = 78, normalized size = 1.00

method	result
default	$\frac{\frac{(\sin^5(x))b^2}{5}+\frac{ab(\sin^3(x))}{3}-\frac{2b^2(\sin^3(x))}{3}+a^2\sin(x)-ab\sin(x)+b^2\sin(x)}{b^3} - \frac{a^3\,\text{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)\,b}}\right)}{b^3\sqrt{(a+b)\,b}}$
risch	$-\frac{ie^{ix}a^2}{2b^3} + \frac{3ie^{ix}a}{8b^2} - \frac{5ie^{ix}}{16b} + \frac{ie^{-ix}a^2}{2b^3} - \frac{3ie^{-ix}a}{8b^2} + \frac{5ie^{-ix}}{16b} + \frac{a^3\ln\left(e^{2ix}-\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}\,b^3} - \frac{a^3\ln\left(e^{2ix}+\frac{2i(a+b)e^{ix}}{\sqrt{ab+b}}\right.}{2\sqrt{ab+b^2}\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*sin(x)^5*b^2+1/3*a*b*sin(x)^3-2/3*b^2*sin(x)^3+a^2*sin(x)-a*b*si
n(x)+b^2*sin(x))-a^3/b^3/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 91, normalized size = 1.17

$$\frac{a^3 \log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,b^3}+\frac{3\,b^2\sin(x)^5+5\,(ab-2\,b^2)\sin(x)^3+15\,(a^2-ab+b^2)\sin(x)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqr
t((a + b)*b)*b^3) + 1/15*(3*b^2*sin(x)^5 + 5*(a*b - 2*b^2)*sin(x)^3 + 15*(a
^2 - a*b + b^2)*sin(x))/b^3
```

Fricas [A]
time = 0.42, size = 259, normalized size = 3.32

$$\left[\frac{15\,\sqrt{ab+b^2}\,a^3\log\left(-\frac{b\cos(x)^2+2\,\sqrt{ab+b^2}\,\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+2\,\left(3\,(ab^3+b^4)\cos(x)^4+15\,a^3b+5\,a^2b^2-2\,ab^3+8\,b^4-(5\,a^2b^2+ab^3-4\,b^4)\cos(x)^2\right)\sin(x)}{30\,(ab^4+b^5)}, \frac{15\,\sqrt{-ab-b^2}\,a^3\arctan\left(\frac{\sqrt{-ab-b^2}\,\sin(x)}{a+b}\right)+\left(3\,(ab^3+b^4)\cos(x)^4+15\,a^3b+5\,a^2b^2-2\,ab^3+8\,b^4-(5\,a^2b^2+ab^3-4\,b^4)\cos(x)^2\right)\sin(x)}{15\,(ab^4+b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a*b + b^2)*a^3*log(-(b*cos(x)^2 + 2*sqrt(a*b + b^2)*sin(x) -
 a - 2*b)/(b*cos(x)^2 + a)) + 2*(3*(a*b^3 + b^4)*cos(x)^4 + 15*a^3*b + 5*a^
2*b^2 - 2*a*b^3 + 8*b^4 - (5*a^2*b^2 + a*b^3 - 4*b^4)*cos(x)^2)*sin(x))/(a*
b^4 + b^5), 1/15*(15*sqrt(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*sin(x)/(a
 + b)) + (3*(a*b^3 + b^4)*cos(x)^4 + 15*a^3*b + 5*a^2*b^2 - 2*a*b^3 + 8*b^4
 - (5*a^2*b^2 + a*b^3 - 4*b^4)*cos(x)^2)*sin(x))/(a*b^4 + b^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 96, normalized size = 1.23

$$\frac{a^3\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^3}+\frac{3\,b^4\sin(x)^5+5\,ab^3\sin(x)^3-10\,b^4\sin(x)^3+15\,a^2b^2\sin(x)-15\,ab^3\sin(x)+15\,b^4\sin(x)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] a^3*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^3) + 1/15*(3*b^4*
sin(x)^5 + 5*a*b^3*sin(x)^3 - 10*b^4*sin(x)^3 + 15*a^2*b^2*sin(x) - 15*a*b^
3*sin(x) + 15*b^4*sin(x))/b^5
```

Mupad [B]
time = 0.13, size = 86, normalized size = 1.10

$$\frac{\sin(x)^5}{5\,b}+\sin(x)^3\left(\frac{a+b}{3\,b^2}-\frac{1}{b}\right)+\sin(x)\,\left(\frac{3}{b}+\frac{(a+b)\,\left(\frac{a+b}{b^2}-\frac{3}{b}\right)}{b}\right)+\frac{a^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,1\mathrm{i}}{\sqrt{a+b}}\right)\,1\mathrm{i}}{b^{7/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a + b*cos(x)^2),x)
```

```
[Out] sin(x)^5/(5*b) + sin(x)^3*((a + b)/(3*b^2) - 1/b) + sin(x)*(3/b + ((a + b)*
((a + b)/b^2 - 3/b))/b) + (a^3*atan((b^(1/2)*sin(x)*1i)/(a + b)^(1/2))*1i)/
(b^(7/2)*(a + b)^(1/2))
```

3.29 $\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

```
[Out] -(a-b)*sin(x)/b^2-1/3*sin(x)^3/b+a^2*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^
(5/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3265, 398, 214}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] (a^2*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]) - ((a - b
)*Sin[x])/b^2 - Sin[x]^3/(3*b)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^5(x)}{a+b\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)dx, x, \sin(x)\right)$$
$$= -\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}+\frac{a^2\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{b^2}$$
$$= \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}}-\frac{(a-b)\sin(x)}{b^2}-\frac{\sin^3(x)}{3b}$$

Mathematica [A]
time = 0.21, size = 86, normalized size = 1.54

$$\frac{\frac{6a^2\left(-\log\left(\sqrt{a+b}-\sqrt{b}\,\sin(x)\right)+\log\left(\sqrt{a+b}+\sqrt{b}\,\sin(x)\right)\right)}{\sqrt{a+b}}+3\sqrt{b}\,(-4a+3b)\sin(x)+b^{3/2}\sin(3x)}{12b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] ((6*a^2*(-Log[Sqrt[a + b] - Sqrt[b]*Sin[x]] + Log[Sqrt[a + b] + Sqrt[b]*Sin
[x]]))/Sqrt[a + b] + 3*Sqrt[b]*(-4*a + 3*b)*Sin[x] + b^(3/2)*Sin[3*x])/(12*
b^(5/2))
```

Maple [A]
time = 0.16, size = 50, normalized size = 0.89

method	result	size
default	$-\frac{\frac{b\left(\sin^3(x)\right)}{3}+\sin(x)a-\sin(x)b}{b^2}+\frac{a^2\,\text{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)\,b}}\right)}{b^2\sqrt{(a+b)\,b}}$	50
risch	$\frac{i e^{ix}a}{2b^2}-\frac{3i e^{ix}}{8b}-\frac{i e^{-ix}a}{2b^2}+\frac{3i e^{-ix}}{8b}+\frac{a^2\ln\left(e^{2ix}+\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}\,b^2}-\frac{a^2\ln\left(e^{2ix}-\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}\,b^2}+\frac{\sin(3x)}{12b}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(1/3*b*sin(x)^3+sin(x)*a-sin(x)*b)+1/b^2*a^2/((a+b)*b)^(1/2)*arctanh
(b*sin(x)/((a+b)*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 67, normalized size = 1.20

$$-\frac{a^2 \log\left(\frac{b \sin(x) - \sqrt{(a+b)b}}{b \sin(x) + \sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\, b^2} - \frac{b \sin(x)^3 + 3(a-b)\sin(x)}{3\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sq
rt((a + b)*b)*b^2) - 1/3*(b*sin(x)^3 + 3*(a - b)*sin(x))/b^2
```

Fricas [A]
time = 0.46, size = 191, normalized size = 3.41

$$\left[\frac{3\sqrt{ab+b^2}\, a^2 \log\left(-\frac{b\cos(x)^2 - 2\sqrt{ab+b^2}\sin(x) - a - 2b}{b\cos(x)^2 + a}\right) - 2\left(3a^2b + ab^2 - 2b^3 - (ab^2+b^3)\cos(x)^2\right)\sin(x)}{6(ab^3+b^4)}, -\frac{3\sqrt{-ab-b^2}\, a^2 \arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right) + \left(3a^2b + ab^2 - 2b^3 - (ab^2+b^3)\cos(x)^2\right)\sin(x)}{3(ab^3+b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a*b + b^2)*a^2*log(-(b*cos(x)^2 - 2*sqrt(a*b + b^2)*sin(x) - a
 - 2*b)/(b*cos(x)^2 + a)) - 2*(3*a^2*b + a*b^2 - 2*b^3 - (a*b^2 + b^3)*cos(
x)^2)*sin(x))/(a*b^3 + b^4), -1/3*(3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a*b
- b^2)*sin(x)/(a + b)) + (3*a^2*b + a*b^2 - 2*b^3 - (a*b^2 + b^3)*cos(x)^2)
*sin(x))/(a*b^3 + b^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 65, normalized size = 1.16

$$-\frac{a^2 \arctan\left(\frac{b \sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\, b^2} - \frac{b^2 \sin(x)^3 + 3\, ab \sin(x) - 3\, b^2 \sin(x)}{3\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -a^2*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b^2) - 1/3*(b^2*si
n(x)^3 + 3*a*b*sin(x) - 3*b^2*sin(x))/b^3
```

Mupad [B]
time = 2.31, size = 51, normalized size = 0.91

$$\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{5/2}\,\sqrt{a+b}}-\frac{\sin\left(x\right)^3}{3\,b}-\sin\left(x\right)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a + b*cos(x)^2),x)
```

```
[Out] (a^2*atanh((b^(1/2)*sin(x))/(a + b)^(1/2)))/(b^(5/2)*(a + b)^(1/2)) - sin(x
)^3/(3*b) - sin(x)*((a + b)/b^2 - 2/b)
```

3.30 $\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}+\frac{\sin(x)}{b}$$

```
[Out] sin(x)/b-a*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3265, 396, 214}

$$\frac{\sin(x)}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sin[x]
/b
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^3(x)}{a+b\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx, x, \sin(x)\right)$$
$$= \frac{\sin(x)}{b} - \frac{a\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{b}$$
$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}} + \frac{\sin(x)}{b}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.00

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}} + \frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] -((a*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b])) + Sin[x]
/b
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.87

method	result	size
default	$\frac{\sin(x)}{b} - \frac{a\,\text{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)\,b}}\right)}{b\sqrt{(a+b)\,b}}$	33
risch	$-\frac{i e^{ix}}{2b} + \frac{i e^{-ix}}{2b} + \frac{a\ln\left(e^{2ix} - \frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}} - 1\right)}{2\sqrt{ab+b^2}\,b} - \frac{a\ln\left(e^{2ix} + \frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}} - 1\right)}{2\sqrt{ab+b^2}\,b}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)/b-1/b*a/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 50, normalized size = 1.32

$$\frac{a\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,b} + \frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqrt(
(a + b)*b)*b) + sin(x)/b
```

Fricas [A]
time = 0.44, size = 134, normalized size = 3.53

$$\left[\frac{\sqrt{ab+b^2}\,a\log\left(-\frac{b\cos\left(x\right)^2+2\sqrt{ab+b^2}\,\sin\left(x\right)-a-2\,b}{b\cos\left(x\right)^2+a}\right)+2\left(ab+b^2\right)\sin\left(x\right)}{2\left(ab^2+b^3\right)},\ \frac{\sqrt{-ab-b^2}\,a\arctan\left(\frac{\sqrt{-ab-b^2}\,\sin\left(x\right)}{a+b}\right)+\left(ab+b^2\right)\sin\left(x\right)}{ab^2+b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*b + b^2)*a*log(-(b*cos(x)^2 + 2*sqrt(a*b + b^2)*sin(x) - a - 2
*b)/(b*cos(x)^2 + a)) + 2*(a*b + b^2)*sin(x))/(a*b^2 + b^3), (sqrt(-a*b - b
^2)*a*arctan(sqrt(-a*b - b^2)*sin(x)/(a + b)) + (a*b + b^2)*sin(x))/(a*b^2
+ b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.39, size = 41, normalized size = 1.08

$$\frac{a\arctan\left(\frac{b\sin\left(x\right)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b}+\frac{\sin\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] a*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b) + sin(x)/b
```

Mupad [B]
time = 0.10, size = 30, normalized size = 0.79

$$\frac{\sin\left(x\right)}{b}-\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sin\left(x\right)}{\sqrt{a+b}}\right)}{b^{3/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a + b*cos(x)^2),x)
```

```
[Out] sin(x)/b - (a*atanh((b^(1/2)*sin(x))/(a + b)^(1/2)))/(b^(3/2)*(a + b)^(1/2)
)
```

3.31 $\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}}$$

```
[Out] arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))/b^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3265, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Cos[x]^2),x]
[Out] ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b])
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.72

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)\,b}}\right)}{\sqrt{(a+b)\,b}}$	21
risch	$\frac{\ln\left(e^{2ix}+\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}}-\frac{\ln\left(e^{2ix}-\frac{2i(a+b)e^{ix}}{\sqrt{ab+b^2}}-1\right)}{2\sqrt{ab+b^2}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 39, normalized size = 1.34

$$-\frac{\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/sqrt((a
 + b)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(21) = 42$.

time = 0.43, size = 95, normalized size = 3.28

$$\left[\frac{\log\left(-\frac{b\cos(x)^2-2\sqrt{ab+b^2}\sin(x)-a-2b}{b\cos(x)^2+a}\right)}{2\sqrt{ab+b^2}},-\frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\sin(x)}{a+b}\right)}{ab+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(x)^2 - 2*sqrt(a*b + b^2)*sin(x) - a - 2*b)/(b*cos(x)^2 + a
))/sqrt(a*b + b^2), -sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^2)*sin(x)/(a + b
))/(a*b + b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 55508 vs. $2(27) = 54$.
time = 69.00, size = 55508, normalized size = 1914.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)**2),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x/2) - 1) + log(tan(x/2) + 1)), Eq(a, 0) & Eq(b, 0
)), (sin(x)/a, Eq(b, 0)), (tan(x/2)/(2*b) + 1/(2*b*tan(x/2)), Eq(a, -b)), (
-13*a**6*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-sqrt(-a
/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**7*b*sqrt(-a/
(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2
*sqrt(-a*b)/(a + b)) - 130*a**6*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-
a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 24*a**6
*b*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(
a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 858*a**5*b**3*sqrt(-a/(a + b)
+ b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a
*b)/(a + b)) + 416*a**5*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqr
t(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 858*
a**4*b**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 1144*a**4*b**3*sqrt(-a*b)*sqrt(-a/
(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2
*sqrt(-a*b)/(a + b)) - 858*a**3*b**5*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-
a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 858*a**
2*b**6*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
+ b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 1144*a**2*b**5*sqrt(-a*b)*sqrt(-a/(a
+ b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sq
rt(-a*b)/(a + b)) - 130*a*b**7*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(
a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 416*a*b**6*sq
```

```
rt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b
) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 2*b**8*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))
 + 24*b**7*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*s
qrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) + 13*a**6*b*sqrt(-a/(a
+ b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) +
2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**7*b*sqrt(-a/(a + b) + b/(a + b) - 2
*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
130*a**6*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(
a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 24*a**6*b*sqrt(-a*b)*sqrt(-a/(
a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*
sqrt(-a*b)/(a + b)) + 858*a**5*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a
*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 416*a**5
*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-
a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 858*a**4*b**4*sqrt(-a/(a +
b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt
(-a*b)/(a + b)) - 1144*a**4*b**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2
*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) -
858*a**3*b**5*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(
a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 858*a**2*b**6*sqrt(-a/(a + b)
+ b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a
*b)/(a + b)) + 1144*a**2*b**5*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sq
rt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 130
*a*b**7*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
 + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 416*a*b**6*sqrt(-a*b)*sqrt(-a/(a + b
) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(
-a*b)/(a + b)) + 2*b**8*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))
*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 24*b**7*sqrt(-a*b)*s
qrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a +
 b) + 2*sqrt(-a*b)/(a + b))) + 11*a**6*b*sqrt(-a/(a + b) + b/(a + b) + 2*sq
rt(-a*b)/(a + b))*log(-sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))
+ tan(x/2))/(2*a**7*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*s
qrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 130*a**6*b**2*sqrt(-a/
(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2
*sqrt(-a*b)/(a + b)) - 24*a**6*b*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2
*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) +
858*a**5*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(
a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 416*a**5*b**2*sqrt(-a*b)*sqrt(
-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) - 858*a**4*b**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqr
t(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 1144
*a**4*b**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*s
qrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 858*a**3*b**5*sqrt(-a/
(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2
*sqrt(-a*b)/(a + b)) + 858*a**2*b**6*sqrt(-a/(a...
```

Giac [**A**]
time = 0.40, size = 31, normalized size = 1.07

$$-\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(b*sin(x)/sqrt(-a*b - b^2))/sqrt(-a*b - b^2)
```

Mupad [**B**]
time = 0.09, size = 21, normalized size = 0.72

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\ \sin(x)}{\sqrt{a+b}}\right)}{\sqrt{b}\ \sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*cos(x)^2),x)
```

```
[Out] atanh((b^(1/2)*sin(x))/(a + b)^(1/2))/(b^(1/2)*(a + b)^(1/2))
```

3.32 $\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}(\sin(x))}{a}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

```
[Out] arctanh(sin(x))/a-arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3265, 400, 212, 214}

$$\frac{\tanh^{-1}(\sin(x))}{a}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Cos[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/a - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[x])/Sqrt[a + b]])/(a*Sqrt
[a + b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+b-bx^2)}\,dx, x, \sin(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(41) = 82.

time = 0.16, size = 93, normalized size = 2.27

$$\frac{-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{\sqrt{b}\left(\log\left(\sqrt{a+b}-\sqrt{b}\,\sin(x)\right)-\log\left(\sqrt{a+b}+\sqrt{b}\,\sin(x)\right)\right)}{\sqrt{a+b}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Cos[x]^2),x]
```

```
[Out] (-2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] + (Sqrt[b]*(Log[S
qrt[a + b] - Sqrt[b]*Sin[x]] - Log[Sqrt[a + b] + Sqrt[b]*Sin[x]]))/Sqrt[a +
 b])/(2*a)
```

Maple [A]

time = 0.13, size = 47, normalized size = 1.15

method	result
default	$\frac{\ln(\sin(x)+1)}{2a} - \frac{b\,\text{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)\,b}}\right)}{a\sqrt{(a+b)\,b}} - \frac{\ln(\sin(x)-1)}{2a}$
risch	$-\frac{\ln(e^{ix}-i)}{a} + \frac{\ln(e^{ix}+i)}{a} + \frac{\sqrt{(a+b)\,b}\,\ln\left(e^{2ix}-\frac{2i\sqrt{(a+b)\,b}\,e^{ix}}{b}-1\right)}{2(a+b)a} - \frac{\sqrt{(a+b)\,b}\,\ln\left(e^{2ix}+\frac{2i\sqrt{(a+b)\,b}}{b}\right.}{2(a+b)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*ln(sin(x)+1)-1/a*b/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))-
1/2/a*ln(sin(x)-1)
```

Maxima [A]
time = 0.48, size = 64, normalized size = 1.56

$$\frac{b \log\left(\frac{b \sin(x) - \sqrt{(a+b)b}}{b \sin(x) + \sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,a} + \frac{\log(\sin(x)+1)}{2a} - \frac{\log(\sin(x)-1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqrt(
(a + b)*b)*a) + 1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

Fricas [A]
time = 0.42, size = 119, normalized size = 2.90

$$\left[\frac{\sqrt{\frac{b}{a+b}} \log\left(-\frac{b\cos(x)^2 + 2(a+b)\sqrt{\frac{b}{a+b}}\sin(x) - a - 2b}{b\cos(x)^2 + a}\right) + \log(\sin(x)+1) - \log(-\sin(x)+1)}{2a}, \frac{2\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\sin(x)\right) + \log(\sin(x)+1) - \log(-\sin(x)+1)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log(-(b*cos(x)^2 + 2*(a + b)*sqrt(b/(a + b))*sin(x) -
 a - 2*b)/(b*cos(x)^2 + a)) + log(sin(x) + 1) - log(-sin(x) + 1))/a, 1/2*(2
*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x)) + log(sin(x) + 1) - log(-
sin(x) + 1))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.39

$$\frac{b \arctan\left(\frac{b \sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a} + \frac{\log(\sin(x)+1)}{2a} - \frac{\log(-\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a) + 1/2*log(sin(x) +
 1)/a - 1/2*log(-sin(x) + 1)/a
```

Mupad [B]
time = 2.50, size = 414, normalized size = 10.10

$$\frac{\operatorname{atanh}(\sin(x))}{a}+\frac{\operatorname{atan}\left(\frac{\frac{\left(2b^3\sin(x)+\frac{\left(2a^2b^2-\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{b(a+b)}\,1\mathrm{i}}{a^2+ba}+\frac{\left(2b^3\sin(x)-\frac{\left(2a^2b^2+\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{b(a+b)}\,1\mathrm{i}}{a^2+ba}}{\frac{\left(2b^3\sin(x)+\frac{\left(2a^2b^2-\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{a^2+ba}-\frac{\left(2b^3\sin(x)-\frac{\left(2a^2b^2+\frac{\sin(x)\left(8a^3b^2+16a^2b^3\right)\sqrt{b(a+b)}}{4\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{2\left(a^2+ba\right)}\right)\sqrt{b(a+b)}}{a^2+ba}}\right)\sqrt{b(a+b)}\,1\mathrm{i}}{a^2+ba}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*cos(x)^2)),x)
```

```
[Out] atanh(sin(x))/a + (atan((((2*b^3*sin(x) + ((2*a^2*b^2 - (sin(x)*(16*a^2*b^3
 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*
b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*b^3*sin(x) - ((2*a^2*b^2
 + (sin(x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*
(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2))/(((2*b^
3*sin(x) + ((2*a^2*b^2 - (sin(x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2)
)/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2))/(
a*b + a^2) - ((2*b^3*sin(x) - ((2*a^2*b^2 + (sin(x)*(16*a^2*b^3 + 8*a^3*b^2
)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(
b*(a + b))^(1/2))/(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2)
```

3.33 $\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{(a-2b)\tanh^{-1}(\sin(x))}{2a^2}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{\sec(x)\tan(x)}{2a}$$

```
[Out] 1/2*(a-2*b)*arctanh(sin(x))/a^2+b^(3/2)*arctanh(sin(x)*b^(1/2)/(a+b)^(1/2))
/a^2/(a+b)^(1/2)+1/2*sec(x)*tan(x)/a
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3265, 425, 536, 212, 214}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}}+\frac{(a-2b)\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Sin[x]])/(2*a^2) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sin[x])/Sqr
t[a + b]])/(a^2*Sqrt[a + b]) + (Sec[x]*Tan[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)}\,dx, x, \sin(x)\right)\\
&= \frac{\sec(x)\tan(x)}{2a} + \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \sin(x)\right)}{2a}\\
&= \frac{\sec(x)\tan(x)}{2a} + \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{2a^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{a^2}\\
&= \frac{(a-2b)\tanh^{-1}(\sin(x))}{2a^2} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}} + \frac{\sec(x)\tan(x)}{2a}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 152 vs. 2(59) = 118.
time = 0.43, size = 152, normalized size = 2.58

$$\frac{-2(a-2b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(a-2b)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\frac{2b^{3/2}\log\left(\sqrt{a+b}-\sqrt{b}\,\sin(x)\right)}{\sqrt{a+b}}+\frac{2b^{3/2}\log\left(\sqrt{a+b}+\sqrt{b}\,\sin(x)\right)}{\sqrt{a+b}}+\frac{a}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2}-\frac{a}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Cos[x]^2),x]
```

```
[Out] (-2*(a - 2*b)*Log[Cos[x/2] - Sin[x/2]] + 2*(a - 2*b)*Log[Cos[x/2] + Sin[x/2
]] - (2*b^(3/2)*Log[Sqrt[a + b] - Sqrt[b]*Sin[x]])/Sqrt[a + b] + (2*b^(3/2)
*Log[Sqrt[a + b] + Sqrt[b]*Sin[x]])/Sqrt[a + b] + a/(Cos[x/2] - Sin[x/2])^2
 - a/(Cos[x/2] + Sin[x/2])^2)/(4*a^2)
```

Maple [A]
time = 0.20, size = 82, normalized size = 1.39

method	result
default	$-\frac{1}{4a(\sin(x)+1)}+\frac{(a-2b)\ln(\sin(x)+1)}{4a^2}+\frac{b^2\,\mathrm{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)\,b}}\right)}{a^2\sqrt{(a+b)\,b}}-\frac{1}{4a(\sin(x)-1)}+\frac{(-a+2b)\ln(\sin(x)-1)}{4a^2}$
risch	$-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2 a}-\frac{\ln(e^{ix}-i)}{2a}+\frac{b\ln(e^{ix}-i)}{a^2}+\frac{\ln(e^{ix}+i)}{2a}-\frac{b\ln(e^{ix}+i)}{a^2}+\frac{\sqrt{(a+b)\,b}\;b\ln\left(e^{2ix}+\frac{2i\sqrt{(a+b)\,b}\;e^{ix}}{b}-\right.}{2(a+b)a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/(sin(x)+1)+1/4*(a-2*b)/a^2*ln(sin(x)+1)+b^2/a^2/((a+b)*b)^(1/2)*arct
anh(b*sin(x)/((a+b)*b)^(1/2))-1/4/a/(sin(x)-1)+1/4/a^2*(-a+2*b)*ln(sin(x)-1
)
```

Maxima [A]
time = 0.49, size = 92, normalized size = 1.56

$$-\frac{b^2\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,a^2}+\frac{(a-2\,b)\log(\sin(x)+1)}{4\,a^2}-\frac{(a-2\,b)\log(\sin(x)-1)}{4\,a^2}-\frac{\sin(x)}{2\left(a\sin(x)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sq
rt((a + b)*b)*a^2) + 1/4*(a - 2*b)*log(sin(x) + 1)/a^2 - 1/4*(a - 2*b)*log(
sin(x) - 1)/a^2 - 1/2*sin(x)/(a*sin(x)^2 - a)
```

Fricas [A]
time = 0.48, size = 186, normalized size = 3.15

$$\left[\frac{2\,b\sqrt{\frac{b}{a+b}}\cos(x)^2\log\left(-\frac{b\cos(x)^2-2\,(a+b)\sqrt{\frac{b}{a+b}}\sin(x)-a-2\,b}{b\cos(x)^2+a}\right)+(a-2\,b)\cos(x)^2\log(\sin(x)+1)-(a-2\,b)\cos(x)^2\log(-\sin(x)+1)+2\,a\sin(x)}{4\,a^2\cos(x)^2},\,-\frac{4\,b\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\sin(x)\right)\cos(x)^2-(a-2\,b)\cos(x)^2\log(\sin(x)+1)+(a-2\,b)\cos(x)^2\log(-\sin(x)+1)-2\,a\sin(x)}{4\,a^2\cos(x)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(b/(a + b))*cos(x)^2*log(-(b*cos(x)^2 - 2*(a + b)*sqrt(b/(a +
 b))*sin(x) - a - 2*b)/(b*cos(x)^2 + a)) + (a - 2*b)*cos(x)^2*log(sin(x) +
1) - (a - 2*b)*cos(x)^2*log(-sin(x) + 1) + 2*a*sin(x))/(a^2*cos(x)^2), -1/4
```

```
*(4*b*sqrt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x))*cos(x)^2 - (a - 2*b)
*cos(x)^2*log(sin(x) + 1) + (a - 2*b)*cos(x)^2*log(-sin(x) + 1) - 2*a*sin(x
))/(a^2*cos(x)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**3/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.41, size = 85, normalized size = 1.44

$$-\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2} + \frac{(a-2\,b)\log\left(\sin(x)+1\right)}{4\,a^2} - \frac{(a-2\,b)\log\left(-\sin(x)+1\right)}{4\,a^2} - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^2) + 1/4*(a - 2*
b)*log(sin(x) + 1)/a^2 - 1/4*(a - 2*b)*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/((
sin(x)^2 - 1)*a)
```

Mupad [B]
time = 2.53, size = 483, normalized size = 8.19

$$-\frac{a^2\sin(x)+a^2\,\mathrm{atanh}(\sin(x))-2\,b^2\,\mathrm{atanh}(\sin(x))+a\,b\sin(x)-a\,b\,\mathrm{atanh}(\sin(x))-a^2\,\mathrm{atanh}(\sin(x))\sin(x)^2+2\,b^2\,\mathrm{atanh}(\sin(x))\sin(x)^2+a\,b\,\mathrm{atanh}(\sin(x))\sin(x)^2+\mathrm{atan}\left(\frac{b^5\sin(x)\sqrt{b^4+a\,b^3}\,8\mathrm{i}-a\sin(x)\,{\left(b^4+a\,b^3\right)}^{3/2}\,4\mathrm{i}-b\sin(x)\,{\left(b^4+a\,b^3\right)}^{3/2}\,8\mathrm{i}+a\,b^4\sin(x)\sqrt{b^4+a\,b^3}\,12\mathrm{i}+a^4\,b\sin(x)\sqrt{b^4+a\,b^3}\,1\mathrm{i}+a^2\,b^3\sin(x)\sqrt{b^4+a\,b^3}\,1\mathrm{i}-a^3\,b^2\sin(x)\sqrt{b^4+a\,b^3}\,2\mathrm{i}}{-a^5\,b^2+a^4\,b^3+5\,a^3\,b^4+3\,a^2\,b^5}\right)\sqrt{b^4+a\,b^3}\,2\mathrm{i}-\mathrm{atan}\left(\frac{b^5\sin(x)\sqrt{b^4+a\,b^3}\,8\mathrm{i}-a\sin(x)\,{\left(b^4+a\,b^3\right)}^{3/2}\,4\mathrm{i}-b\sin(x)\,{\left(b^4+a\,b^3\right)}^{3/2}\,8\mathrm{i}+a\,b^4\sin(x)\sqrt{b^4+a\,b^3}\,12\mathrm{i}+a^4\,b\sin(x)\sqrt{b^4+a\,b^3}\,1\mathrm{i}+a^2\,b^3\sin(x)\sqrt{b^4+a\,b^3}\,1\mathrm{i}-a^3\,b^2\sin(x)\sqrt{b^4+a\,b^3}\,2\mathrm{i}}{-a^5\,b^2+a^4\,b^3+5\,a^3\,b^4+3\,a^2\,b^5}\right)\sin(x)^2\sqrt{b^4+a\,b^3}\,2\mathrm{i}}{2\,a^3\sin(x)^2-2\,a^3+2\,b\,a^2\sin(x)^2-2\,b\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a + b*cos(x)^2)),x)
```

```
[Out] -(a^2*sin(x) + a^2*atanh(sin(x)) - 2*b^2*atanh(sin(x)) + atan((b^5*sin(x)*(
a*b^3 + b^4)^(1/2)*8i - a*sin(x)*(a*b^3 + b^4)^(3/2)*4i - b*sin(x)*(a*b^3 +
 b^4)^(3/2)*8i + a*b^4*sin(x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b*sin(x)*(a*b^3
 + b^4)^(1/2)*1i + a^2*b^3*sin(x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*sin(x)*(
a*b^3 + b^4)^(1/2)*2i)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^5*b^2))*(a*b^3
+ b^4)^(1/2)*2i + a*b*sin(x) - a*b*atanh(sin(x)) - a^2*atanh(sin(x))*sin(x)
^2 + 2*b^2*atanh(sin(x))*sin(x)^2 - atan((b^5*sin(x)*(a*b^3 + b^4)^(1/2)*8i
 - a*sin(x)*(a*b^3 + b^4)^(3/2)*4i - b*sin(x)*(a*b^3 + b^4)^(3/2)*8i + a*b^
4*sin(x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b*sin(x)*(a*b^3 + b^4)^(1/2)*1i + a^
2*b^3*sin(x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*sin(x)*(a*b^3 + b^4)^(1/2)*2i
)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^5*b^2))*sin(x)^2*(a*b^3 + b^4)^(1/2)
*2i + a*b*atanh(sin(x))*sin(x)^2)/(2*a^3*sin(x)^2 - 2*a^2*b - 2*a^3 + 2*a^2
*b*sin(x)^2)
```

3.34 $\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=90

$$\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\sin(x))}{8a^3}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a}$$

```
[Out] 1/8*(3*a^2-4*a*b+8*b^2)*arctanh(sin(x))/a^3-b^(5/2)*arctanh(sin(x)*b^(1/2)/
(a+b)^(1/2))/a^3/(a+b)^(1/2)+1/8*(3*a-4*b)*sec(x)*tan(x)/a^2+1/4*sec(x)^3*t
an(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3265, 425, 541, 536, 212, 214}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}}+\frac{(3a-4b)\tan(x)\sec(x)}{8a^2}+\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\sin(x))}{8a^3}+\frac{\tan(x)\sec^3(x)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] ((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Sin[x]])/(8*a^3) - (b^(5/2)*ArcTanh[(Sqrt[
b]*Sin[x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]) + ((3*a - 4*b)*Sec[x]*Tan[x])/(8
*a^2) + (Sec[x]^3*Tan[x])/(4*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
```

```
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^5(x)}{a+b\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)^3\,(a+b-bx^2)}\,dx, x, \sin(x)\right)\\
&= \frac{\sec^3(x)\tan(x)}{4a} + \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \sin(x)\right)}{4a}\\
&= \frac{(3a-4b)\sec(x)\tan(x)}{8a^2} + \frac{\sec^3(x)\tan(x)}{4a} + \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2-(3a-4b)bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \sin(x)\right)}{8a^2}\\
&= \frac{(3a-4b)\sec(x)\tan(x)}{8a^2} + \frac{\sec^3(x)\tan(x)}{4a} - \frac{b^3\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \sin(x)\right)}{a^3} + \frac{(3a^2 -
\\
&= \frac{(3a^2-4ab+8b^2)\tanh^{-1}(\sin(x))}{8a^3} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}} + \frac{(3a-4b)\sec(x)\tan(x}{8a^2}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(90) = 180.

time = 1.34, size = 215, normalized size = 2.39

$$\frac{-2(3a^2-4ab+8b^2)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(3a^2-4ab+8b^2)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{8b^{5/2}\log\left(\sqrt{a+b}-\sqrt{b}\sin(x)\right)}{\sqrt{a+b}}-\frac{8b^{5/2}\log\left(\sqrt{a+b}+\sqrt{b}\sin(x)\right)}{\sqrt{a+b}}+\frac{a^2}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^4}-\frac{a^2}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4}+\frac{a(-3a+4b)}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{a(-3a+4b)}{-1+\sin(x)}}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5/(a + b*Cos[x]^2),x]
```

```
[Out] (-2*(3*a^2 - 4*a*b + 8*b^2)*Log[Cos[x/2] - Sin[x/2]] + 2*(3*a^2 - 4*a*b + 8
*b^2)*Log[Cos[x/2] + Sin[x/2]] + (8*b^(5/2)*Log[Sqrt[a + b] - Sqrt[b]*Sin[x
]])/Sqrt[a + b] - (8*b^(5/2)*Log[Sqrt[a + b] + Sqrt[b]*Sin[x]])/Sqrt[a + b]
 + a^2/(Cos[x/2] - Sin[x/2])^4 - a^2/(Cos[x/2] + Sin[x/2])^4 + (a*(-3*a + 4
*b))/(Cos[x/2] + Sin[x/2])^2 + (a*(-3*a + 4*b))/(-1 + Sin[x]))/(16*a^3)
```

Maple [A]
time = 0.25, size = 137, normalized size = 1.52

method	result
default	$-\frac{1}{16a(\sin(x)+1)^2}-\frac{3a-4b}{16a^2(\sin(x)+1)}+\frac{(3a^2-4ab+8b^2)\ln(\sin(x)+1)}{16a^3}-\frac{b^3\operatorname{arctanh}\left(\frac{b\sin(x)}{\sqrt{(a+b)b}}\right)}{a^3\sqrt{(a+b)b}}+\frac{1}{16a(\sin(x)-1)^2}-$
risch	$-\frac{i(3a\,e^{7ix}-4b\,e^{7ix}+11a\,e^{5ix}-4b\,e^{5ix}-11a\,e^{3ix}+4b\,e^{3ix}-3a\,e^{ix}+4b\,e^{ix})}{4(e^{2ix}+1)^4a^2}-\frac{3\ln(e^{ix}-i)}{8a}+\frac{b\ln(e^{ix}-i)}{2a^2}-\frac{\ln(e^{ix}-i)b^2}{a^3}+\frac{3\ln(e}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a/(sin(x)+1)^2-1/16*(3*a-4*b)/a^2/(sin(x)+1)+1/16*(3*a^2-4*a*b+8*b^2)
/a^3*ln(sin(x)+1)-b^3/a^3/((a+b)*b)^(1/2)*arctanh(b*sin(x)/((a+b)*b)^(1/2))
+1/16/a/(sin(x)-1)^2-1/16*(3*a-4*b)/a^2/(sin(x)-1)+1/16/a^3*(-3*a^2+4*a*b-8
*b^2)*ln(sin(x)-1)
```

Maxima [A]
time = 0.48, size = 145, normalized size = 1.61

$$\frac{b^3\log\left(\frac{b\sin(x)-\sqrt{(a+b)b}}{b\sin(x)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}\,a^3}-\frac{(3\,a-4\,b)\sin(x)^3-(5\,a-4\,b)\sin(x)}{8\left(a^2\sin(x)^4-2\,a^2\sin(x)^2+a^2\right)}+\frac{(3\,a^2-4\,ab+8\,b^2)\log(\sin(x)+1)}{16\,a^3}-\frac{(3\,a^2-4\,ab+8\,b^2)\log(\sin(x)-1)}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log((b*sin(x) - sqrt((a + b)*b))/(b*sin(x) + sqrt((a + b)*b)))/(sqr
t((a + b)*b)*a^3) - 1/8*((3*a - 4*b)*sin(x)^3 - (5*a - 4*b)*sin(x))/(a^2*si
```

```
n(x)^4 - 2*a^2*sin(x)^2 + a^2) + 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(sin(x) +
1)/a^3 - 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(sin(x) - 1)/a^3
```

Fricas [A]
time = 0.45, size = 270, normalized size = 3.00

$$\left[\frac{8\,b^{2}\sqrt{\frac{b}{a+b}}\cos\left(x\right)^{4}\log\left(-\frac{b\cos\left(x\right)^{2}+2\,(a+b)\sqrt{\frac{b}{a+b}}\sin\left(x\right)-a-2\,b}{b\cos\left(x\right)^{2}+a}\right)+\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\cos\left(x\right)^{4}\log\left(\sin\left(x\right)+1\right)-\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\cos\left(x\right)^{4}\log\left(-\sin\left(x\right)+1\right)+2\left(\left(3\,a^{2}-4\,ab\right)\cos\left(x\right)^{2}+2\,a^{2}\right)\sin\left(x\right)}{16\,a^{3}\cos\left(x\right)^{4}},\ \frac{16\,b^{2}\sqrt{-\frac{b}{a+b}}\arctan\left(\sqrt{-\frac{b}{a+b}}\sin\left(x\right)\right)\cos\left(x\right)^{4}+\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\cos\left(x\right)^{4}\log\left(\sin\left(x\right)+1\right)-\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\cos\left(x\right)^{4}\log\left(-\sin\left(x\right)+1\right)+2\left(\left(3\,a^{2}-4\,ab\right)\cos\left(x\right)^{2}+2\,a^{2}\right)\sin\left(x\right)}{16\,a^{3}\cos\left(x\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*b^2*sqrt(b/(a + b))*cos(x)^4*log(-(b*cos(x)^2 + 2*(a + b)*sqrt(b/(
a + b))*sin(x) - a - 2*b)/(b*cos(x)^2 + a)) + (3*a^2 - 4*a*b + 8*b^2)*cos(x
)^4*log(sin(x) + 1) - (3*a^2 - 4*a*b + 8*b^2)*cos(x)^4*log(-sin(x) + 1) + 2
*((3*a^2 - 4*a*b)*cos(x)^2 + 2*a^2)*sin(x))/(a^3*cos(x)^4), 1/16*(16*b^2*sq
rt(-b/(a + b))*arctan(sqrt(-b/(a + b))*sin(x))*cos(x)^4 + (3*a^2 - 4*a*b +
8*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a^2 - 4*a*b + 8*b^2)*cos(x)^4*log(-sin
(x) + 1) + 2*((3*a^2 - 4*a*b)*cos(x)^2 + 2*a^2)*sin(x))/(a^3*cos(x)^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(x\right)}{a+b\cos^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**5/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.41, size = 127, normalized size = 1.41

$$\frac{b^{3}\arctan\left(\frac{b\sin\left(x\right)}{\sqrt{-ab-b^{2}}}\right)}{\sqrt{-ab-b^{2}}\,a^{3}}+\frac{\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\log\left(\sin\left(x\right)+1\right)}{16\,a^{3}}-\frac{\left(3\,a^{2}-4\,ab+8\,b^{2}\right)\log\left(-\sin\left(x\right)+1\right)}{16\,a^{3}}-\frac{3\,a\sin\left(x\right)^{3}-4\,b\sin\left(x\right)^{3}-5\,a\sin\left(x\right)+4\,b\sin\left(x\right)}{8\left(\sin\left(x\right)^{2}-1\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*a^3) + 1/16*(3*a^2
- 4*a*b + 8*b^2)*log(sin(x) + 1)/a^3 - 1/16*(3*a^2 - 4*a*b + 8*b^2)*log(-si
n(x) + 1)/a^3 - 1/8*(3*a*sin(x)^3 - 4*b*sin(x)^3 - 5*a*sin(x) + 4*b*sin(x))
/((sin(x)^2 - 1)^2*a^2)
```

Mupad [B]
time = 2.64, size = 969, normalized size = 10.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5*(a + b*cos(x)^2)),x)
```

```
[Out] (5*a^3*sin(x) + atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i - a*sin(x)*(a*b^5
 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b^6*sin(x)*(a*b^5
 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i + a^2*b^5*sin(x)*(
a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)*40i + a^4*b^3*s
in(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6)^(1/2)*6i)/(40*
a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))*(a*b^5 + b^6)^(
1/2)*8i - 3*a^3*sin(x)^3 + 3*a^3*atanh(sin(x)) + 8*b^3*atanh(sin(x)) - 4*a*
b^2*sin(x) + a^2*b*sin(x) - atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i - a*s
in(x)*(a*b^5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b^6*s
in(x)*(a*b^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i + a^2*
b^5*sin(x)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)*40i
 + a^4*b^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6)^(1
/2)*6i)/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))*sin
(x)^2*(a*b^5 + b^6)^(1/2)*16i + atan((b^7*sin(x)*(a*b^5 + b^6)^(1/2)*128i -
 a*sin(x)*(a*b^5 + b^6)^(3/2)*64i - b*sin(x)*(a*b^5 + b^6)^(3/2)*128i + a*b
^6*sin(x)*(a*b^5 + b^6)^(1/2)*192i + a^6*b*sin(x)*(a*b^5 + b^6)^(1/2)*9i +
a^2*b^5*sin(x)*(a*b^5 + b^6)^(1/2)*64i + a^3*b^4*sin(x)*(a*b^5 + b^6)^(1/2)
*40i + a^4*b^3*sin(x)*(a*b^5 + b^6)^(1/2)*25i - a^5*b^2*sin(x)*(a*b^5 + b^6
)^(1/2)*6i)/(40*a^3*b^7 + 65*a^4*b^6 + 19*a^5*b^5 + 3*a^6*b^4 + 9*a^7*b^3))
*sin(x)^4*(a*b^5 + b^6)^(1/2)*8i - 6*a^3*atanh(sin(x))*sin(x)^2 + 3*a^3*ata
nh(sin(x))*sin(x)^4 - 16*b^3*atanh(sin(x))*sin(x)^2 + 8*b^3*atanh(sin(x))*s
in(x)^4 + 4*a*b^2*sin(x)^3 + a^2*b*sin(x)^3 + 4*a*b^2*atanh(sin(x)) - a^2*b
*atanh(sin(x)) - 8*a*b^2*atanh(sin(x))*sin(x)^2 + 2*a^2*b*atanh(sin(x))*sin
(x)^2 + 4*a*b^2*atanh(sin(x))*sin(x)^4 - a^2*b*atanh(sin(x))*sin(x)^4)/(8*a
^4*sin(x)^4 - 16*a^4*sin(x)^2 + 8*a^3*b + 8*a^4 - 16*a^3*b*sin(x)^2 + 8*a^3
*b*sin(x)^4)
```

3.35 $\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=87

$$\frac{(8a^2-4ab+3b^2)\,x}{8b^3}+\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}}-\frac{(4a-3b)\cos(x)\sin(x)}{8b^2}+\frac{\cos^3(x)\sin(x)}{4b}$$

```
[Out] 1/8*(8*a^2-4*a*b+3*b^2)*x/b^3-1/8*(4*a-3*b)*cos(x)*sin(x)/b^2+1/4*cos(x)^3*
sin(x)/b+a^(5/2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/b^3/(a+b)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3266, 481, 592, 536, 209, 211}

$$\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}}+\frac{x(8a^2-4ab+3b^2)}{8b^3}-\frac{(4a-3b)\sin(x)\cos(x)}{8b^2}+\frac{\sin(x)\cos^3(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) + (a^(5/2)*ArcTan[(Sqrt[a + b]*Cot[x])/
Sqrt[a]])/(b^3*Sqrt[a + b]) - ((4*a - 3*b)*Cos[x]*Sin[x])/(8*b^2) + (Cos[x]
^3*Sin[x])/(4*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3\,(a+(a+b)x^2)}\,dx, x, \cot(x)\right)$$

$$= \frac{\cos^3(x)\sin(x)}{4b} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(-a+3b)x^2)}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \cot(x)\right)}{4b}$$

$$= -\frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(x)\sin(x)}{4b} + \frac{\text{Subst}\left(\int \frac{a(4a-3b)+(-4a^2+ab-3b^2)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \text{co}\right.}{8b^2}$$

$$= -\frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(x)\sin(x)}{4b} + \frac{a^3\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b^3} - \frac{(8}{}$$

$$= \frac{(8a^2-4ab+3b^2)\,x}{8b^3} + \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}} - \frac{(4a-3b)\cos(x)\sin(x)}{8b^2} + \frac{\cos^3(}{}$$

Mathematica [A]
time = 0.24, size = 76, normalized size = 0.87

$$\frac{4(8a^2-4ab+3b^2)\,x-\frac{32a^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-8(a-b)b\sin(2x)+b^2\sin(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (4*(8*a^2 - 4*a*b + 3*b^2)*x - (32*a^(5/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a +
 b]])/Sqrt[a + b] - 8*(a - b)*b*Sin[2*x] + b^2*Sin[4*x])/(32*b^3)
```

Maple [A]
time = 0.19, size = 92, normalized size = 1.06

method	result
default	$\frac{\frac{\left(-\frac{1}{2}ab+\frac{3}{8}b^2\right)\left(\tan^3(x)\right)+\left(-\frac{1}{2}ab+\frac{5}{8}b^2\right)\tan(x)}{\left(1+\tan^2(x)\right)^2}+\frac{\left(8a^2-4ab+3b^2\right)\arctan(\tan(x))}{8}}{b^3}-\frac{a^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{b^3\sqrt{(a+b)\,a}}$
risch	$\frac{x\,a^2}{b^3}-\frac{ax}{2b^2}+\frac{3x}{8b}+\frac{ie^{2ix}a}{8b^2}-\frac{ie^{2ix}}{8b}-\frac{ie^{-2ix}a}{8b^2}+\frac{ie^{-2ix}}{8b}+\frac{\sqrt{-(a+b)\,a}\;a^2\ln\left(e^{2ix}+\frac{2i\sqrt{-(a+b)\,a}+2a+b}{b}\right)}{2(a+b)b^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(((-1/2*a*b+3/8*b^2)*tan(x)^3+(-1/2*a*b+5/8*b^2)*tan(x))/(tan(x)^2+1)
^2+1/8*(8*a^2-4*a*b+3*b^2)*arctan(tan(x)))-1/b^3*a^3/((a+b)*a)^(1/2)*arctan
(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 97, normalized size = 1.11

$$-\frac{a^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}-\frac{(4\,a-3\,b)\tan\left(x\right)^3+(4\,a-5\,b)\tan\left(x\right)}{8\left(b^2\tan\left(x\right)^4+2\,b^2\tan\left(x\right)^2+b^2\right)}+\frac{(8\,a^2-4\,ab+3\,b^2)x}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -a^3*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3) - 1/8*((4*a - 3
*b)*tan(x)^3 + (4*a - 5*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(x)^2 + b^2) +
1/8*(8*a^2 - 4*a*b + 3*b^2)*x/b^3
```

Fricas [A]
time = 0.44, size = 273, normalized size = 3.14

$$\left[\frac{2\,a^{2}\sqrt{-\frac{a}{a+b}}\,\log\left(\frac{(8\,a^{2}+8\,ab+b^{2})\cos(x)^{4}-2\,(4\,a^{2}+3\,ab)\cos(x)^{2}+4\left((2\,a^{2}+3\,ab+b^{2})\cos(x)^{3}-(a^{2}+ab)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\sin(x)+a^{2}}{b^{2}\cos(x)^{4}+2\,ab\cos(x)^{2}+a^{2}}\right)+(8\,a^{2}-4\,ab+3\,b^{2})x+\left(2\,b^{2}\cos(x)^{3}-(4\,ab-3\,b^{2})\cos(x)\right)\sin(x)}{8\,b^{3}},\ \frac{4\,a^{2}\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^{2}-a\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(x)\sin(x)}\right)+(8\,a^{2}-4\,ab+3\,b^{2})x+\left(2\,b^{2}\cos(x)^{3}-(4\,ab-3\,b^{2})\cos(x)\right)\sin(x)}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2
 + 3*a*b)*cos(x)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a^2 + a*b)*cos(x)
)*sqrt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) + (
8*a^2 - 4*a*b + 3*b^2)*x + (2*b^2*cos(x)^3 - (4*a*b - 3*b^2)*cos(x))*sin(x)
)/b^3, 1/8*(4*a^2*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(
a/(a + b))/(a*cos(x)*sin(x))) + (8*a^2 - 4*a*b + 3*b^2)*x + (2*b^2*cos(x)^3
 - (4*a*b - 3*b^2)*cos(x))*sin(x))/b^3]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 104, normalized size = 1.20

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^{2}+ab}}\right)\right)a^{3}}{\sqrt{a^{2}+ab}\,b^{3}}+\frac{(8\,a^{2}-4\,ab+3\,b^{2})x}{8\,b^{3}}-\frac{4\,a\tan(x)^{3}-3\,b\tan(x)^{3}+4\,a\tan(x)-5\,b\tan(x)}{8\left(\tan(x)^{2}+1\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a^3/(sqrt
(a^2 + a*b)*b^3) + 1/8*(8*a^2 - 4*a*b + 3*b^2)*x/b^3 - 1/8*(4*a*tan(x)^3 -
3*b*tan(x)^3 + 4*a*tan(x) - 5*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

Mupad [B]
time = 2.69, size = 1036, normalized size = 11.91

$$-\frac{\frac{\tan(x)^{3}(4\,a-3\,b)}{8\,b^{2}}+\frac{\tan(x)(4\,a-5\,b)}{8\,b^{2}}}{\tan(x)^{4}+2\tan(x)^{2}+1}-\frac{\mathrm{atan}\left(\frac{63\,a^{4}\tan(x)}{64\left(\frac{63\,a^{4}}{64}-\frac{81\,a^{3}b}{256}+\frac{27\,a^{2}b^{2}}{256}-\frac{35\,a^{5}}{32\,b}+\frac{5\,a^{6}}{4\,b^{2}}\right)}-\frac{81\,a^{3}\tan(x)}{256\left(\frac{27\,a^{2}b}{256}-\frac{81\,a^{3}}{256}+\frac{63\,a^{4}}{64\,b}-\frac{35\,a^{5}}{32\,b^{2}}+\frac{5\,a^{6}}{4\,b^{3}}\right)}-\frac{35\,a^{5}\tan(x)}{32\left(\frac{63\,a^{4}b}{64}-\frac{35\,a^{5}}{32}+\frac{27\,a^{2}b^{3}}{256}-\frac{81\,a^{3}b^{2}}{256}+\frac{5\,a^{6}}{4\,b}\right)}+\frac{5\,a^{6}\tan(x)}{4\left(\frac{5\,a^{6}}{4}-\frac{35\,a^{5}b}{32}+\frac{63\,a^{4}b^{2}}{64}-\frac{81\,a^{3}b^{3}}{256}+\frac{27\,a^{2}b^{4}}{256}\right)}+\frac{27\,a^{2}\tan(x)}{256\left(\frac{27\,a^{2}}{256}-\frac{81\,a^{3}}{256\,b}+\frac{63\,a^{4}}{64\,b^{2}}-\frac{35\,a^{5}}{32\,b^{3}}+\frac{5\,a^{6}}{4\,b^{4}}\right)}\right)\left(a^{2}\,8\mathrm{i}-a\,b\,4\mathrm{i}+b^{2}\,3\mathrm{i}\right)1\mathrm{i}}{8\,b^{3}}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-a^{5}(a+b)}\left(\frac{\sqrt{-a^{5}(a+b)}\left(\frac{2\,a^{4}b^{6}-\frac{a^{3}b^{7}}{2}+\frac{3\,a^{2}b^{8}}{2}}{2\,b^{6}}-\frac{\tan(x)(512\,a^{3}b^{6}+256\,a^{2}b^{7})\sqrt{-a^{5}(a+b)}}{128\,b^{4}(b^{4}+a\,b^{3})}\right)}{2\,(b^{4}+a\,b^{3})}-\frac{\tan(x)(128\,a^{7}-64\,a^{6}b+64\,a^{5}b^{2}-24\,a^{4}b^{3}+9\,a^{3}b^{4})}{64\,b^{4}}\right)1\mathrm{i}}{b^{4}+a\,b^{3}}-\frac{\sqrt{-a^{5}(a+b)}\left(\frac{\sqrt{-a^{5}(a+b)}\left(\frac{2\,a^{4}b^{6}-\frac{a^{3}b^{7}}{2}+\frac{3\,a^{2}b^{8}}{2}}{2\,b^{6}}+\frac{\tan(x)(512\,a^{3}b^{6}+256\,a^{2}b^{7})\sqrt{-a^{5}(a+b)}}{128\,b^{4}(b^{4}+a\,b^{3})}\right)}{2\,(b^{4}+a\,b^{3})}+\frac{\tan(x)(128\,a^{7}-64\,a^{6}b+64\,a^{5}b^{2}-24\,a^{4}b^{3}+9\,a^{3}b^{4})}{64\,b^{4}}\right)1\mathrm{i}}{b^{4}+a\,b^{3}}}{\frac{\sqrt{-a^{5}(a+b)}\left(\frac{\sqrt{-a^{5}(a+b)}\left(\frac{2\,a^{4}b^{6}-\frac{a^{3}b^{7}}{2}+\frac{3\,a^{2}b^{8}}{2}}{2\,b^{6}}-\frac{\tan(x)(512\,a^{3}b^{6}+256\,a^{2}b^{7})\sqrt{-a^{5}(a+b)}}{128\,b^{4}(b^{4}+a\,b^{3})}\right)}{2\,(b^{4}+a\,b^{3})}-\frac{\tan(x)(128\,a^{7}-64\,a^{6}b+64\,a^{5}b^{2}-24\,a^{4}b^{3}+9\,a^{3}b^{4})}{64\,b^{4}}\right)}{b^{4}+a\,b^{3}}-\frac{-a^{8}+\frac{5\,a^{7}b}{4}-\frac{3\,a^{6}b^{2}}{4}+\frac{9\,a^{5}b^{3}}{32}}{b^{6}}+\frac{\sqrt{-a^{5}(a+b)}\left(\frac{\sqrt{-a^{5}(a+b)}\left(\frac{2\,a^{4}b^{6}-\frac{a^{3}b^{7}}{2}+\frac{3\,a^{2}b^{8}}{2}}{2\,b^{6}}+\frac{\tan(x)(512\,a^{3}b^{6}+256\,a^{2}b^{7})\sqrt{-a^{5}(a+b)}}{128\,b^{4}(b^{4}+a\,b^{3})}\right)}{2\,(b^{4}+a\,b^{3})}+\frac{\tan(x)(128\,a^{7}-64\,a^{6}b+64\,a^{5}b^{2}-24\,a^{4}b^{3}+9\,a^{3}b^{4})}{64\,b^{4}}\right)}{b^{4}+a\,b^{3}}}\right)\sqrt{-a^{5}(a+b)}\,1\mathrm{i}}{b^{4}+a\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a + b*cos(x)^2),x)
```

```
[Out] - ((tan(x)^3*(4*a - 3*b))/(8*b^2) + (tan(x)*(4*a - 5*b))/(8*b^2))/(2*tan(x)
^2 + tan(x)^4 + 1) - (atan((63*a^4*tan(x))/(64*((63*a^4)/64 - (81*a^3*b)/25
6 + (27*a^2*b^2)/256 - (35*a^5)/(32*b) + (5*a^6)/(4*b^2))) - (81*a^3*tan(x)
)/(256*((27*a^2*b)/256 - (81*a^3)/256 + (63*a^4)/(64*b) - (35*a^5)/(32*b^2)
 + (5*a^6)/(4*b^3))) - (35*a^5*tan(x))/(32*((63*a^4*b)/64 - (35*a^5)/32 + (
27*a^2*b^3)/256 - (81*a^3*b^2)/256 + (5*a^6)/(4*b))) + (5*a^6*tan(x))/(4*((
5*a^6)/4 - (35*a^5*b)/32 + (27*a^2*b^4)/256 - (81*a^3*b^3)/256 + (63*a^4*b^
2)/64)) + (27*a^2*tan(x))/(256*((27*a^2)/256 - (81*a^3)/(256*b) + (63*a^4)/
(64*b^2) - (35*a^5)/(32*b^3) + (5*a^6)/(4*b^4))))*(a^2*8i - a*b*4i + b^2*3i
)*1i)/(8*b^3) - (atan((((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^
2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(256*a^2*b^7 + 512*a^
3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) -
(tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4
))*1i)/(a*b^3 + b^4) - ((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^
2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(256*a^2*b^7 + 512*a^
3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) +
(tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4
))*1i)/(a*b^3 + b^4))/(((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^
2*b^8)/2 - (a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(256*a^2*b^7 + 512*a^
3*b^6)*(-a^5*(a + b))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) -
(tan(x)*(128*a^7 - 64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4
)))/(a*b^3 + b^4) - ((5*a^7*b)/4 - a^8 + (9*a^5*b^3)/32 - (3*a^6*b^2)/4)/b^
6 + ((-a^5*(a + b))^(1/2)*(((-a^5*(a + b))^(1/2)*(((3*a^2*b^8)/2 - (a^3*b^7
)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(256*a^2*b^7 + 512*a^3*b^6)*(-a^5*(a + b
))^(1/2))/(128*b^4*(a*b^3 + b^4))))/(2*(a*b^3 + b^4)) + (tan(x)*(128*a^7 -
64*a^6*b + 9*a^3*b^4 - 24*a^4*b^3 + 64*a^5*b^2))/(64*b^4)))/(a*b^3 + b^4)))
*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4)
```

3.36 $\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=60

$$-\frac{(2a-b)x}{2b^2}-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}}+\frac{\cos(x)\sin(x)}{2b}$$

```
[Out] -1/2*(2*a-b)*x/b^2+1/2*cos(x)*sin(x)/b-a^(3/2)*arctan(cot(x)*(a+b)^(1/2)/a^
(1/2))/b^2/(a+b)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3266, 481, 536, 209, 211}

$$-\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}+\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -1/2*((2*a - b)*x)/b^2 - (a^(3/2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(b^
2*Sqrt[a + b]) + (Cos[x]*Sin[x])/(2*b)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
```

```
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2\,(a+(a+b)x^2)}\,dx, x, \cot(x)\right) \\
&= \frac{\cos(x)\sin(x)}{2b} - \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \cot(x)\right)}{2b} \\
&= \frac{\cos(x)\sin(x)}{2b} - \frac{a^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b^2} + \frac{(2a-b)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(x)\right)}{2b^2} \\
&= -\frac{(2a-b)x}{2b^2} - \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}} + \frac{\cos(x)\sin(x)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 0.87

$$
\frac{2(-2a+b)x + \frac{4a^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + b\sin(2x)}{4b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] (2*(-2*a + b)*x + (4*a^(3/2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a +
 b] + b*Sin[2*x])/(4*b^2)
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.98

method	result
default	$-\frac{-\frac{b\tan(x)}{2\left(1+\tan^2(x)\right)}+\frac{(2a-b)\arctan(\tan(x))}{2}}{b^2}+\frac{a^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{b^2\sqrt{(a+b)\,a}}$
risch	$-\frac{ax}{b^2}+\frac{x}{2b}-\frac{i e^{2ix}}{8b}+\frac{i e^{-2ix}}{8b}+\frac{\sqrt{-(a+b)\,a}\,a\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)\,a}-2a-b}{b}\right)}{2(a+b)b^2}-\frac{\sqrt{-(a+b)\,a}\,a\ln\left(e^2\right.}{2($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/2*b*tan(x)/(tan(x)^2+1)+1/2*(2*a-b)*arctan(tan(x)))+1/b^2*a^2/((
a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 54, normalized size = 0.90

$$\frac{a^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}-\frac{(2\,a-b)x}{2\,b^2}+\frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] a^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2) - 1/2*(2*a - b)*
x/b^2 + 1/2*tan(x)/(b*tan(x)^2 + b)
```

Fricas [A]
time = 0.46, size = 213, normalized size = 3.55

$$\left[\frac{2\,b\cos(x)\sin(x)+a\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2-4\left((2\,a^2+3\,ab+b^2)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)-2\,(2\,a-b)x}{4\,b^2},\ \frac{b\cos(x)\sin(x)-a\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(x)\sin(x)}\right)-(2\,a-b)x}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*cos(x)*sin(x) + a*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a
^2 + a*b)*cos(x))*sqrt(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(
x)^2 + a^2)) - 2*(2*a - b)*x)/b^2, 1/2*(b*cos(x)*sin(x) - a*sqrt(a/(a + b))
```

```
*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(a/(a + b))/(a*cos(x)*sin(x))) - (
2*a - b)*x)/b^2]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 72, normalized size = 1.20

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}\,b^2}-\frac{(2\,a-b)x}{2\,b^2}+\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a^2/(sqrt(
a^2 + a*b)*b^2) - 1/2*(2*a - b)*x/b^2 + 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

Mupad [B]
time = 2.61, size = 291, normalized size = 4.85

$$-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-b^2\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{b^2\,\sin(2\,x)}{2}+a\,b\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{a\,b\,\sin(2\,x)}{2}+\mathrm{atan}\left(\frac{a\,\sin(x)\,{\left(-a^4-b\,a^3\right)}^{3/2}\,8{}\mathrm{i}+b\,\sin(x)\,{\left(-a^4-b\,a^3\right)}^{3/2}\,4{}\mathrm{i}+a^5\,\sin(x)\,\sqrt{-a^4-b\,a^3}\,8{}\mathrm{i}-a^2\,b^3\,\sin(x)\,\sqrt{-a^4-b\,a^3}\,2{}\mathrm{i}+a^3\,b^2\,\sin(x)\,\sqrt{-a^4-b\,a^3}\,1{}\mathrm{i}+a\,b^4\,\sin(x)\,\sqrt{-a^4-b\,a^3}\,1{}\mathrm{i}+a^4\,b\,\sin(x)\,\sqrt{-a^4-b\,a^3}\,12{}\mathrm{i}}{3\,\cos(x)\,a^5\,b^2+5\,\cos(x)\,a^4\,b^3+\cos(x)\,a^3\,b^4-\cos(x)\,a^2\,b^5}\right)\,\sqrt{-a^4-b\,a^3}\,2{}\mathrm{i}}{2\,b^3+2\,a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a + b*cos(x)^2),x)
```

```
[Out] -(2*a^2*atan(sin(x)/cos(x)) - b^2*atan(sin(x)/cos(x)) + atan((a*sin(x)*(- a
^3*b - a^4)^(3/2)*8i + b*sin(x)*(- a^3*b - a^4)^(3/2)*4i + a^5*sin(x)*(- a^
3*b - a^4)^(1/2)*8i - a^2*b^3*sin(x)*(- a^3*b - a^4)^(1/2)*2i + a^3*b^2*sin
(x)*(- a^3*b - a^4)^(1/2)*1i + a*b^4*sin(x)*(- a^3*b - a^4)^(1/2)*1i + a^4*
b*sin(x)*(- a^3*b - a^4)^(1/2)*12i)/(a^3*b^4*cos(x) - a^2*b^5*cos(x) + 5*a^
4*b^3*cos(x) + 3*a^5*b^2*cos(x)))*(- a^3*b - a^4)^(1/2)*2i - (b^2*sin(2*x))
/2 + a*b*atan(sin(x)/cos(x)) - (a*b*sin(2*x))/2)/(2*a*b^2 + 2*b^3)
```

3.37 $\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=38

$$\frac{x}{b}+\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}$$

```
[Out] x/b+arctan(cot(x)*(a+b)^(1/2)/a^(1/2))*a^(1/2)/b/(a+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3250, 3260, 211}

$$\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] x/b + (Sqrt[a]*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(b*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a+b\cos^2(x)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\cos^2(x)}\,dx}{b}$$
$$= \frac{x}{b} + \frac{a\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{b}$$
$$= \frac{x}{b} + \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{b\sqrt{a+b}}$$

Mathematica [A]
time = 0.09, size = 36, normalized size = 0.95

$$\frac{x - \frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Cos[x]^2),x]
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/Sqrt[a + b])/b
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.89

method	result
default	$\frac{\arctan(\tan(x))}{b} - \frac{a\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{b\sqrt{(a+b)\,a}}$
risch	$\frac{x}{b} + \frac{\sqrt{-(a+b)\,a}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-(a+b)\,a}+2a+b}{b}\right)}{2(a+b)b} - \frac{\sqrt{-(a+b)\,a}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-(a+b)\,a}-2a-b}{b}\right)}{2(a+b)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/b*arctan(tan(x))-1/b*a/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.82

$$-\frac{a\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) + x/b
```

Fricas [A]
time = 0.42, size = 183, normalized size = 4.82

$$\left[\frac{\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2+4\,\left((2\,a^2+3\,ab+b^2)\cos(x)^3-(a^2+ab)\cos(x)\right)\sqrt{-\frac{a}{a+b}}\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)+4\,x}{4\,b}, \frac{\sqrt{\frac{a}{a+b}}\arctan\left(\frac{\left((2\,a+b)\cos(x)^2-a\right)\sqrt{\frac{a}{a+b}}}{2\,a\cos(x)\sin(x)}\right)+2\,x}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(x)^3 - (a^2 + a*b)*cos(x))*sqrt
(-a/(a + b))*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) + 4*x)/b,
 1/2*(sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(x)^2 - a)*sqrt(a/(a + b))/(
a*cos(x)*sin(x))) + 2*x)/b]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 48, normalized size = 1.26

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*a/(sqrt(a
^2 + a*b)*b) + x/b
```

Mupad [B]
time = 2.55, size = 425, normalized size = 11.18

$$\frac{x}{b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,a^3\,\tan(x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}\,1{}\mathrm{i}}{b^2+a\,b}+\frac{\left(2\,a^3\,\tan(x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}\,1{}\mathrm{i}}{b^2+a\,b}}{\frac{\left(2\,a^3\,\tan(x)-\frac{\left(2\,a^2\,b^2-\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}}{b^2+a\,b}-\frac{\left(2\,a^3\,\tan(x)+\frac{\left(2\,a^2\,b^2+\frac{\tan(x)\,\left(16\,a^3\,b^2+8\,a^2\,b^3\right)\,\sqrt{-a\,\left(a+b\right)}}{4\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}}{2\,\left(b^2+a\,b\right)}\right)\,\sqrt{-a\,\left(a+b\right)}}{b^2+a\,b}}\right)\,\sqrt{-a\,\left(a+b\right)}\,1{}\mathrm{i}}{b^2+a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*cos(x)^2),x)
```

```
[Out] x/b - (atan((((2*a^3*tan(x) - ((2*a^2*b^2 - (tan(x)*(8*a^2*b^3 + 16*a^3*b^2
)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))
*(-a*(a + b))^(1/2)*1i)/(a*b + b^2) + ((2*a^3*tan(x) + ((2*a^2*b^2 + (tan(x
)*(8*a^2*b^3 + 16*a^3*b^2)*(-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b)
)^(1/2))/(2*(a*b + b^2)))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2))/(((2*a^3*tan(
x) - ((2*a^2*b^2 - (tan(x)*(8*a^2*b^3 + 16*a^3*b^2)*(-a*(a + b))^(1/2))/(4*
(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))*(-a*(a + b))^(1/2))/(a*b
 + b^2) - ((2*a^3*tan(x) + ((2*a^2*b^2 + (tan(x)*(8*a^2*b^3 + 16*a^3*b^2)*(
-a*(a + b))^(1/2))/(4*(a*b + b^2)))*(-a*(a + b))^(1/2))/(2*(a*b + b^2)))*(-
a*(a + b))^(1/2))/(a*b + b^2)))*(-a*(a + b))^(1/2)*1i)/(a*b + b^2)
```

3.38 $\int \frac{1}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

```
[Out] -arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3260, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-1),x]
```

```
[Out] -(ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 29, normalized size = 0.97

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-1),x]
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]]/(Sqrt[a]*Sqrt[a + b])
```

Maple [A]
time = 0.00, size = 21, normalized size = 0.70

method	result	s
default	$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{\sqrt{(a+b)\,a}}$	2
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}+\frac{\ln\left(\mathrm{e}^{2ix}-\frac{2ia^2+2iab-2a\sqrt{-a^2-ab}-b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.48, size = 20, normalized size = 0.67

$$\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="maxima")
[Out] arctan(a*tan(x)/sqrt((a + b)*a))/sqrt((a + b)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.
time = 0.41, size = 163, normalized size = 5.43

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2+4\,\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)}{4\,(a^2+ab)},\ -\frac{\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\,\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)}{2\,\sqrt{a^2+ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a
*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2))/(a^2 + a*b), -1/2*arctan(1/2*((
2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))/sqrt(a^2 + a*b)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10924 vs. $2(29) = 58$.
time = 19.21, size = 10924, normalized size = 364.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-2*tan(x/
2)/(b*(tan(x/2)**2 - 1)), Eq(a, 0)), (-tan(x/2)/(2*b) + 1/(2*b*tan(x/2)), E
q(a, -b)), (a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-s
qrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*sqrt
(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
 + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-
a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a**3*
sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a + b) + b/
(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/
(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqr
t(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqrt(-a*b)*sqrt
(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
 + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)
/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2
))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a
+ b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a
+ b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b
))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(
-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b)
+ 2*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*
b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*
sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) + a**3*sqrt(-a/(a + b) + b/(a + b)
 + 2*sqrt(-a*b)/(a + b))*log(-sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a
 + b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a
```

```
/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) +
2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*s
qrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10
*a**2*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a +
 b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b) + b/(a +
 b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a +
 b)) + 8*a*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a +
b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - a**3*sqrt(-a/(a
+ b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))*log(sqrt(-a/(a + b) + b/(a + b) -
2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*s
qrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10
*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
 + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) +
b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b
)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b
))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(
a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*
sqrt(-a*b)/(a + b)) + 8*a*b**2*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*s
qrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))) - 1
0*a**2*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*log(-sqrt(-a/(
a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/(2*a**4*sqrt(-a/(a +
 b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqr
t(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a
+ b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 8*a**3*sqrt(-a*
b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/
(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/(a + b) + b/(a + b)
- 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b))
 + 2*a*b**3*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a
+ b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + 8*a*b**2*sqrt(-a*b)*sqrt(-a/(a +
 b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqr
t(-a*b)/(a + b))) + 10*a**2*b*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a
 + b))*log(sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) + tan(x/2))/
(2*a**4*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b)
 + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**3*b*sqrt(-a/(a + b) + b/(a + b
) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b
)) - 8*a**3*sqrt(-a*b)*sqrt(-a/(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*
sqrt(-a/(a + b) + b/(a + b) + 2*sqrt(-a*b)/(a + b)) - 10*a**2*b**2*sqrt(-a/
(a + b) + b/(a + b) - 2*sqrt(-a*b)/(a + b))*sqrt(-a/(a + b) + b/(a + b) + 2
*sqrt(-a*b)/(a + b)) + 2*a*b**3*sqrt(-a/(a + b)...
```

Giac [A]
time = 0.41, size = 37, normalized size = 1.23

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a\tan(x)}{\sqrt{a^2 + ab}}\right)}{\sqrt{a^2 + ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))/sqrt(a^2 +
 a*b)
```

Mupad [B]
time = 0.00, size = 24, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{a\tan(x)}{\sqrt{a^2+b\,a}}\right)}{\sqrt{a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2),x)
```

```
[Out] atan((a*tan(x))/(a*b + a^2)^(1/2))/(a*b + a^2)^(1/2)
```

3.39 $\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{a}$$

```
[Out] b*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(1/2)+tan(x)/a
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3266, 464, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Cos[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]) + Tan[x]/a
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1+x^2}{x^2\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)\\ &= \frac{\tan(x)}{a} + \frac{b\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{a}\\ &= \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{\tan(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 38, normalized size = 1.03

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Cos[x]^2),x]
[Out] -((b*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*Sqrt[a + b])) + Tan[x]/
a
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.89

method	result
default	$\frac{\tan(x)}{a} - \frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{a\sqrt{(a+b)\,a}}$
risch	$\frac{2i}{a(e^{2ix}+1)} - \frac{b\ln\left(e^{2ix} + \frac{-2ia^2-2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,a} + \frac{b\ln\left(e^{2ix} + \frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-}}{b\sqrt{-a^2-ab}}\right.}{2\sqrt{-a^2-ab}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] tan(x)/a-1/a*b/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.49, size = 32, normalized size = 0.86

$$-\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{\tan(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a) + tan(x)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(29) = 58.
time = 0.42, size = 216, normalized size = 5.84

$$\left[-\frac{\sqrt{-a^2-ab}\,b\cos(x)\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+3ab)\cos(x)^2-4((2a+b)\cos(x)^3-a\cos(x))\sqrt{-a^2-ab}\sin(x)+a^2}{b^2\cos(x)^4+2ab\cos(x)^2+a^2}\right)-4(a^2+ab)\sin(x)}{4(a^3+a^2b)\cos(x)},\ \frac{\sqrt{a^2+ab}\,b\arctan\left(\frac{(2a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\cos(x)+2(a^2+ab)\sin(x)}{2(a^3+a^2b)\cos(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4
*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)
*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*(a^2 + a*b)*sin(x
))/((a^3 + a^2*b)*cos(x)), 1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos
(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) + 2*(a^2 + a*b)*sin(x))/
((a^3 + a^2*b)*cos(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(x)}{a+b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.42, size = 36, normalized size = 0.97

$$-\frac{b\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,a}+\frac{\tan(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(a*tan(x)/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*a) + tan(x)/a
```

Mupad [B]
time = 2.38, size = 30, normalized size = 0.81

$$\frac{\tan(x)}{a}-\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{3/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*cos(x)^2)),x)
```

```
[Out] tan(x)/a - (b*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(3/2)*(a + b)^(1/2))
```

3.40 $\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}$$

```
[Out] -b^2*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(1/2)+(a-b)*tan(x)/a^
2+1/3*tan(x)^3/a
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3266, 472, 211}

$$-\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^2*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b])) + ((a -
 b)*Tan[x])/a^2 + Tan[x]^3/(3*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(x)}{a+b\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \cot(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{a-b}{a^2x^2}+\frac{b^2}{a^2\left(a+(a+b)x^2\right)}\right)dx, x, \cot(x)\right)\\
&= \frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}-\frac{b^2\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \cot(x)\right)}{a^2}\\
&= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(a-b)\tan(x)}{a^2}+\frac{\tan^3(x)}{3a}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 55, normalized size = 0.98

$$
\frac{b^2\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{(2a-3b+a\sec^2(x))\tan(x)}{3a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Cos[x]^2),x]
[Out] (b^2*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + ((2*a - 
3*b + a*Sec[x]^2)*Tan[x])/(3*a^2)
```

Maple [A]
time = 0.18, size = 49, normalized size = 0.88

method	result
default	$\frac{\frac{a(\tan^3(x))}{3}+\tan(x)a-\tan(x)b}{a^2}+\frac{b^2\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{a^2\sqrt{(a+b)\,a}}$
risch	$-\frac{2i(3b\,e^{4ix}-6a\,e^{2ix}+6b\,e^{2ix}-2a+3b)}{3(e^{2ix}+1)^3a^2}-\frac{b^2\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,a^2}+\frac{b^2\ln\left(e^{2ix}-\frac{2ia^2+2i}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/3*a*tan(x)^3+tan(x)*a-tan(x)*b)+b^2/a^2/((a+b)*a)^(1/2)*arctan(a*t
an(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 48, normalized size = 0.86

$$\frac{b^2 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2} + \frac{a\tan\left(x\right)^3 + 3\left(a-b\right)\tan\left(x\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^2) + 1/3*(a*tan(x)^
3 + 3*(a - b)*tan(x))/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(46) = 92.
time = 0.46, size = 276, normalized size = 4.93

$$\left[-\frac{3\sqrt{-a^2-ab}\,b^2\cos\left(x\right)^3\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+3\,ab\right)\cos(x)^2+4\left(\left(2\,a+b\right)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)-4\left(a^3+a^2b+\left(2\,a^3-a^2b-3\,ab^2\right)\cos\left(x\right)^2\right)\sin\left(x\right)}{12\left(a^4+a^3b\right)\cos\left(x\right)^3}, -\frac{3\sqrt{a^2+ab}\,b^2\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)\cos\left(x\right)^3-2\left(a^3+a^2b+\left(2\,a^3-a^2b-3\,ab^2\right)\cos\left(x\right)^2\right)\sin\left(x\right)}{6\left(a^4+a^3b\right)\cos\left(x\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4
 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2
 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*(a^3 + a^2
*b + (2*a^3 - a^2*b - 3*a*b^2)*cos(x)^2)*sin(x))/((a^4 + a^3*b)*cos(x)^3),
-1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 +
 a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + a^2*b + (2*a^3 - a^2*b - 3*a*b^2)
*cos(x)^2)*sin(x))/((a^4 + a^3*b)*cos(x)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(x\right)}{a+b\cos^2\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**4/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.41, size = 71, normalized size = 1.27

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}\,a^2} + \frac{a^2\tan\left(x\right)^3+3\,a^2\tan\left(x\right)-3\,ab\tan\left(x\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^2/(sqrt(
a^2 + a*b)*a^2) + 1/3*(a^2*tan(x)^3 + 3*a^2*tan(x) - 3*a*b*tan(x))/a^3
```

Mupad [B]
time = 2.31, size = 51, normalized size = 0.91

$$\frac{\tan(x)^3}{3\,a}-\tan(x)\,\left(\frac{a+b}{a^2}-\frac{2}{a}\right)+\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{5/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a + b*cos(x)^2)),x)
```

```
[Out] tan(x)^3/(3*a) - tan(x)*((a + b)/a^2 - 2/a) + (b^2*atan((a^(1/2)*tan(x))/(a
 + b)^(1/2)))/(a^(5/2)*(a + b)^(1/2))
```

3.41 $\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=79

$$\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(a^2-ab+b^2)\tan(x)}{a^3}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a}$$

```
[Out] b^3*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(7/2)/(a+b)^(1/2)+(a^2-a*b+b^2)*ta
n(x)/a^3+1/3*(2*a-b)*tan(x)^3/a^2+1/5*tan(x)^5/a
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3266, 472, 211}

$$\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}}+\frac{(2a-b)\tan^3(x)}{3a^2}+\frac{(a^2-ab+b^2)\tan(x)}{a^3}+\frac{\tan^5(x)}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]) + ((a^2 -
a*b + b^2)*Tan[x])/a^3 + ((2*a - b)*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3266

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[x^m*((a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1)),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^6(x)}{a+b\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6\,(a+(a+b)x^2)}\,dx, x, \cot(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{ax^6} + \frac{2a-b}{a^2x^4} + \frac{a^2-ab+b^2}{a^3x^2} + \frac{b^3}{a^3\,(-a-(a+b)x^2)}\right)\,dx, x, \cot(x)\right)$$
$$= \frac{(a^2-ab+b^2)\tan(x)}{a^3} + \frac{(2a-b)\tan^3(x)}{3a^2} + \frac{\tan^5(x)}{5a} - \frac{b^3\text{Subst}\left(\int \frac{1}{-a-(a+b)x^2}\,dx, x, \text{co}\right.}{a^3}$$
$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}} + \frac{(a^2-ab+b^2)\tan(x)}{a^3} + \frac{(2a-b)\tan^3(x)}{3a^2} + \frac{\tan^5(x)}{5a}$$

Mathematica [A]
time = 0.38, size = 80, normalized size = 1.01

$$-\frac{b^3\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{7/2}\sqrt{a+b}} + \frac{(8a^2-10ab+15b^2+a(4a-5b)\sec^2(x)+3a^2\sec^4(x))\tan(x)}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6/(a + b*Cos[x]^2),x]
```

```
[Out] -((b^3*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(7/2)*Sqrt[a + b])) + ((8*a
^2 - 10*a*b + 15*b^2 + a*(4*a - 5*b)*Sec[x]^2 + 3*a^2*Sec[x]^4)*Tan[x])/(15
*a^3)
```

Maple [A]
time = 0.24, size = 78, normalized size = 0.99

method	result
default	$\frac{\frac{(\tan^5(x))a^2}{5}+\frac{2a^2(\tan^3(x))}{3}-\frac{ab(\tan^3(x))}{3}+a^2\tan(x)-ab\tan(x)+b^2\tan(x)}{a^3} - \frac{b^3\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{a^3\sqrt{(a+b)\,a}}$
risch	$\frac{2i(15b^2e^{8ix}-30ab\,e^{6ix}+60b^2e^{6ix}+80a^2e^{4ix}-70ab\,e^{4ix}+90b^2e^{4ix}+40a^2e^{2ix}-50b\,e^{2ix}a+60b^2e^{2ix}+8a^2-10ab+15b^2)}{15a^3(e^{2ix}+1)^5} - \frac{b^3\ln\left(e^{2ix}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6/(a+b*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/5*tan(x)^5*a^2+2/3*a^2*tan(x)^3-1/3*a*b*tan(x)^3+a^2*tan(x)-a*b*ta
n(x)+b^2*tan(x))-b^3/a^3/((a+b)*a)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 74, normalized size = 0.94

$$-\frac{b^3 \arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3} + \frac{3\,a^2\tan(x)^5 + 5\,(2\,a^2 - ab)\tan(x)^3 + 15\,(a^2 - ab + b^2)\tan(x)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b^3*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^3) + 1/15*(3*a^2*t
an(x)^5 + 5*(2*a^2 - a*b)*tan(x)^3 + 15*(a^2 - a*b + b^2)*tan(x))/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.
time = 0.48, size = 348, normalized size = 4.41

$$\left[-\frac{15\sqrt{-a^2-ab}\,b^3\cos(x)^5\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2-4\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right) - 4\left((8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3)\cos(x)^4+3\,a^4+3\,a^3b+(4\,a^4-a^3b-5\,a^2b^2)\cos(x)^2\right)\sin(x)}{60\,(a^5+a^4b)\cos(x)^5},\ \frac{15\sqrt{a^2+ab}\,b^3\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\sqrt{a^2+ab}\cos(x)\sin(x)}\right)\cos(x)^5 + 2\left((8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3)\cos(x)^4+3\,a^4+3\,a^3b+(4\,a^4-a^3b-5\,a^2b^2)\cos(x)^2\right)\sin(x)}{30\,(a^5+a^4b)\cos(x)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 - 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^
2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)) - 4*((8*a^4 -
 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(x)^4 + 3*a^4 + 3*a^3*b + (4*a^4 - a^3*
b - 5*a^2*b^2)*cos(x)^2)*sin(x))/((a^5 + a^4*b)*cos(x)^5), 1/30*(15*sqrt(a^
2 + a*b)*b^3*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*si
n(x)))*cos(x)^5 + 2*((8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(x)^4 + 3*
a^4 + 3*a^3*b + (4*a^4 - a^3*b - 5*a^2*b^2)*cos(x)^2)*sin(x))/((a^5 + a^4*b
)*cos(x)^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(x)}{a + b\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6/(a+b*cos(x)**2),x)
```

```
[Out] Integral(sec(x)**6/(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.41, size = 104, normalized size = 1.32

$$-\frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan(x)}{\sqrt{a^2 + ab}}\right)\right) b^3}{\sqrt{a^2 + ab}\, a^3} + \frac{3\,a^4 \tan(x)^5 + 10\,a^4 \tan(x)^3 - 5\,a^3 b \tan(x)^3 + 15\,a^4 \tan(x) - 15\,a^3 b \tan(x) + 15\,a^2 b^2 \tan(x)}{15\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*cos(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*b^3/(sqrt
(a^2 + a*b)*a^3) + 1/15*(3*a^4*tan(x)^5 + 10*a^4*tan(x)^3 - 5*a^3*b*tan(x)^
3 + 15*a^4*tan(x) - 15*a^3*b*tan(x) + 15*a^2*b^2*tan(x))/a^5
```

Mupad [B]
time = 2.30, size = 84, normalized size = 1.06

$$\frac{\tan(x)^5}{5\,a} - \tan(x)^3 \left(\frac{a+b}{3\,a^2} - \frac{1}{a}\right) + \tan(x) \left(\frac{3}{a} + \frac{(a+b)\,\left(\frac{a+b}{a^2} - \frac{3}{a}\right)}{a}\right) - \frac{b^3 \operatorname{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{a^{7/2}\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6*(a + b*cos(x)^2)),x)
```

```
[Out] tan(x)^5/(5*a) - tan(x)^3*((a + b)/(3*a^2) - 1/a) + tan(x)*(3/a + ((a + b)*
((a + b)/a^2 - 3/a))/a) - (b^3*atan((a^(1/2)*tan(x))/(a + b)^(1/2)))/(a^(7/
2)*(a + b)^(1/2))
```

3.42 $\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{(2a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

```
[Out] -1/2*(2*a+b)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)-1/2*b*c
os(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3263, 12, 3260, 211}

$$-\frac{(2a+b)\text{ArcTan}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(x)\cos(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-2),x]
```

```
[Out] -1/2*((2*a + b)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(3/2
)) - (b*Cos[x]*Sin[x])/(2*a*(a + b)*(a + b*Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
```

```
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b\cos^2(x)\right)^2}\,dx &= -\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}-\frac{\int \frac{-2a-b}{a+b\cos^2(x)}\,dx}{2a(a+b)}\\
&= -\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}+\frac{(2a+b)\int \frac{1}{a+b\cos^2(x)}\,dx}{2a(a+b)}\\
&= -\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}-\frac{(2a+b)\mathrm{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\cot(x)\right)}{2a(a+b)}\\
&= -\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\cos(x)\sin(x)}{2a(a+b)\left(a+b\cos^2(x)\right)}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 70, normalized size = 1.08

$$-\frac{(-2a-b)\mathrm{ArcTan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\sin(2x)}{2a(a+b)(2a+b+b\cos(2x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-2),x]
[Out] -1/2*((-2*a - b)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a^(3/2)*(a + b)^(3/
2)) - (b*Sin[2*x])/(2*a*(a + b)*(2*a + b + b*Cos[2*x]))
```

Maple [A]
time = 0.13, size = 60, normalized size = 0.92

method	result
default	$-\frac{b\tan(x)}{2(a+b)a\left(a\left(\tan^2(x)\right)+a+b\right)}+\frac{(2a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{2(a+b)a\sqrt{(a+b)\,a}}$
risch	$-\frac{i\left(2a\,e^{2ix}+b\,e^{2ix}+b\right)}{(a+b)a\left(b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}+b\right)}-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a\sqrt{-a^2-ab}+b\sqrt{-a^2-ab}}{b\sqrt{-a^2-ab}}\right)}{2\sqrt{-a^2-ab}\,(a+b)}-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2a}{\ldots}\right.}{4\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/(a+b)/a*tan(x)/(a*tan(x)^2+a+b)+1/2*(2*a+b)/(a+b)/a/((a+b)*a)^(1/2)*
arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 72, normalized size = 1.11

$$-\frac{b\tan(x)}{2\left(a^3+2\,a^2b+ab^2+(a^3+a^2b)\tan(x)^2\right)}+\frac{(2\,a+b)\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{2\,\sqrt{(a+b)a}\,(a^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*tan(x)/(a^3 + 2*a^2*b + a*b^2 + (a^3 + a^2*b)*tan(x)^2) + 1/2*(2*a +
 b)*arctan(a*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + a*b))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.
time = 0.45, size = 326, normalized size = 5.02

$$\left[-\frac{4\,(a^2b+ab^2)\cos(x)\sin(x)+\left((2\,ab+b^2)\cos(x)^2+2\,a^2+ab\right)\sqrt{-a^2-ab}\,\log\left(\frac{(8\,a^2+8\,ab+b^2)\cos(x)^4-2\,(4\,a^2+3\,ab)\cos(x)^2+4\,\left((2\,a+b)\cos(x)^3-a\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4+2\,ab\cos(x)^2+a^2}\right)}{8\left(a^5+2\,a^4b+a^3b^2+(a^4b+2\,a^3b^2+a^2b^3)\cos(x)^2\right)},-\frac{2\,(a^2b+ab^2)\cos(x)\sin(x)+\left((2\,ab+b^2)\cos(x)^2+2\,a^2+ab\right)\sqrt{a^2+ab}\,\arctan\left(\frac{(2\,a+b)\cos(x)^2-a}{2\,\sqrt{a^2+ab}\,\cos(x)\sin(x)}\right)}{4\left(a^5+2\,a^4b+a^3b^2+(a^4b+2\,a^3b^2+a^2b^3)\cos(x)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(x)*sin(x) + ((2*a*b + b^2)*cos(x)^2 + 2*a^2 +
a*b)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*
b)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a
^2)/(b^2*cos(x)^4 + 2*a*b*cos(x)^2 + a^2)))/(a^5 + 2*a^4*b + a^3*b^2 + (a^4
*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2), -1/4*(2*(a^2*b + a*b^2)*cos(x)*sin(x)
+ ((2*a*b + b^2)*cos(x)^2 + 2*a^2 + a*b)*sqrt(a^2 + a*b)*arctan(1/2*((2*a +
 b)*cos(x)^2 - a)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^5 + 2*a^4*b + a^3*b^
2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*cos(x)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 69, normalized size = 1.06

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(2a+b)}{2\left(a^2+ab\right)^{\frac{3}{2}}}-\frac{b\tan(x)}{2\left(a\tan(x)^2+a+b\right)\left(a^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(2*a +
 b)/(a^2 + a*b)^(3/2) - 1/2*b*tan(x)/((a*tan(x)^2 + a + b)*(a^2 + a*b))
```

Mupad [B]
time = 2.34, size = 52, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)(2a+b)}{2a^{3/2}(a+b)^{3/2}}-\frac{b\tan(x)}{2a\,(a+b)\,\left(a\tan(x)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^2,x)
```

```
[Out] (atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(2*a + b))/(2*a^(3/2)*(a + b)^(3/2))
- (b*tan(x))/(2*a*(a + b)*(a + b + a*tan(x)^2))
```

3.43 $\int \frac{1}{(a+b\cos^2(x))^3}\,dx$

Optimal. Leaf size=107

$$-\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{b\cos(x)\sin(x)}{4a(a+b)\,(a+b\cos^2(x))^2}-\frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\,(a+b\cos^2(x))}$$

```
[Out] -1/8*(8*a^2+8*a*b+3*b^2)*arctan(cot(x)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(
5/2)-1/4*b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^2-3/8*b*(2*a+b)*cos(x)*sin(
x)/a^2/(a+b)^2/(a+b*cos(x)^2)
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3263, 3252, 12, 3260, 211}

$$-\frac{3b(2a+b)\sin(x)\cos(x)}{8a^2(a+b)^2\,(a+b\cos^2(x))}-\frac{(8a^2+8ab+3b^2)\,\text{ArcTan}\left(\frac{\sqrt{a+b}\,\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}}-\frac{b\sin(x)\cos(x)}{4a(a+b)\,(a+b\cos^2(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-3),x]
```

```
[Out] -1/8*((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Cot[x])/Sqrt[a]])/(a^(5/2
)*(a + b)^(5/2)) - (b*Cos[x]*Sin[x])/(4*a*(a + b)*(a + b*Cos[x]^2)^2) - (3*
b*(2*a + b)*Cos[x]*Sin[x])/(8*a^2*(a + b)^2*(a + b*Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+b\cos^2(x))^3}\,dx = -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{\int \frac{-4a-3b+2b\cos^2(x)}{(a+b\cos^2(x))^2}\,dx}{4a(a+b)}$$

$$= -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)} - \frac{\int \frac{-8a^2-8ab-3b^2}{a+b\cos^2(x)}\,dx}{8a^2(a+b)^2}$$

$$= -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)} + \frac{(8a^2+8ab+3b^2)\int \frac{}{a\text{-}}}{8a^2(a+b)^2}$$

$$= -\frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{3b(2a+b)\cos(x)\sin(x)}{8a^2(a+b)^2\left(a+b\cos^2(x)\right)} - \frac{(8a^2+8ab+3b^2)\,\mathrm{Sub}}{8a}$$

$$= -\frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\cot(x)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}} - \frac{b\cos(x)\sin(x)}{4a(a+b)\left(a+b\cos^2(x)\right)^2} - \frac{3b(2a}{8a^2(a+}$$

Mathematica [A]
time = 0.74, size = 106, normalized size = 0.99

$$\frac{\frac{(8a^2+8ab+3b^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{\sqrt{a}\,b\left(16a^2+16ab+3b^2+3b(2a+b)\cos(2x)\right)\sin(2x)}{(a+b)^2(2a+b+b\cos(2x))^2}}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-3),x]
```

```
[Out] (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + b]])/(a + b)^(5/
2) - (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 + 3*b*(2*a + b)*Cos[2*x])*Sin[2*x]
)/((a + b)^2*(2*a + b + b*Cos[2*x])^2))/(8*a^(5/2))
```

Maple [A]
time = 0.22, size = 117, normalized size = 1.09

method	result
default	$\frac{-\frac{b(8a+5b)\left(\tan^{3}(x)\right)}{8a\left(a^{2}+2ab+b^{2}\right)}-\frac{(8a+3b)b\tan(x)}{8a^{2}(a+b)}}{\left(a(\tan^{2}(x))+a+b\right)^{2}}+\frac{(8a^{2}+8ab+3b^{2})\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)\,a}}\right)}{8(a^{2}+2ab+b^{2})a^{2}\sqrt{(a+b)\,a}}$
risch	$-\frac{i\left(8a^{2}b\,e^{6ix}+8a\,b^{2}e^{6ix}+3b^{3}e^{6ix}+48a^{3}e^{4ix}+72a^{2}b\,e^{4ix}+42a\,b^{2}e^{4ix}+9b^{3}e^{4ix}+40a^{2}b\,e^{2ix}+40a\,b^{2}e^{2ix}+9b^{3}e^{2ix}+6b^{2}a+3b^{3}\right)}{4(a+b)^{2}a^{2}\left(b\,e^{4ix}+4a\,e^{2ix}+2b\,e^{2ix}+b\right)^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tan(x)^3-1/8*(8*a+3*b)/a^2*b/(a+b)*tan(
x))/(a*tan(x)^2+a+b)^2+1/8*(8*a^2+8*a*b+3*b^2)/(a^2+2*a*b+b^2)/a^2/((a+b)*a
)^(1/2)*arctan(a*tan(x)/((a+b)*a)^(1/2))
```

Maxima [A]
time = 0.47, size = 186, normalized size = 1.74

$$\frac{(8\,a^{2}+8\,ab+3\,b^{2})\arctan\left(\frac{a\tan(x)}{\sqrt{(a+b)a}}\right)}{8\,(a^{4}+2\,a^{3}b+a^{2}b^{2})\sqrt{(a+b)a}}-\frac{(8\,a^{2}b+5\,ab^{2})\tan(x)^{3}+(8\,a^{2}b+11\,ab^{2}+3\,b^{3})\tan(x)}{8\left(a^{6}+4\,a^{5}b+6\,a^{4}b^{2}+4\,a^{3}b^{3}+a^{2}b^{4}+(a^{6}+2\,a^{5}b+a^{4}b^{2})\tan(x)^{4}+2\,(a^{6}+3\,a^{5}b+3\,a^{4}b^{2}+a^{3}b^{3})\tan(x)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*arctan(a*tan(x)/sqrt((a + b)*a))/((a^4 + 2*a^3*
b + a^2*b^2)*sqrt((a + b)*a)) - 1/8*((8*a^2*b + 5*a*b^2)*tan(x)^3 + (8*a^2*
b + 11*a*b^2 + 3*b^3)*tan(x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*
b^4 + (a^6 + 2*a^5*b + a^4*b^2)*tan(x)^4 + 2*(a^6 + 3*a^5*b + 3*a^4*b^2 + a
^3*b^3)*tan(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(93) = 186.
time = 0.47, size = 616, normalized size = 5.76

$$\left[-\frac{\left((8\,a^{2}b^{2}+8\,ab^{3}+3\,b^{4})\cos(x)^{4}+8\,a^{4}+8\,a^{3}b+3\,a^{2}b^{2}+2\,(8\,a^{3}b+8\,a^{2}b^{2}+3\,ab^{3})\cos(x)^{2}\right)\sqrt{-a^{2}-ab}\,\log\left(\frac{(8\,a^{2}+8\,ab+b^{2})\cos(x)^{4}-2\,(4\,a^{2}+3\,ab)\cos(x)^{2}+4\left((2\,a+b)\cos(x)^{3}-a\cos(x)\right)\sqrt{-a^{2}-ab}\,\sin(x)+a^{2}}{b^{2}\cos(x)^{4}+2\,ab\cos(x)^{2}+a^{2}}\right)+4\left(3\,(2\,a^{3}b^{2}+3\,a^{2}b^{3}+ab^{4})\cos(x)^{3}+(8\,a^{4}b+13\,a^{3}b^{2}+5\,a^{2}b^{3})\cos(x)\right)\sin(x)}{32\left(a^{8}+3\,a^{7}b+3\,a^{6}b^{2}+a^{5}b^{3}+(a^{6}b^{2}+3\,a^{5}b^{3}+3\,a^{4}b^{4}+a^{3}b^{5})\cos(x)^{4}+2\,(a^{7}b+3\,a^{6}b^{2}+3\,a^{5}b^{3}+a^{4}b^{4})\cos(x)^{2}\right)},\ -\frac{\left((8\,a^{2}b^{2}+8\,ab^{3}+3\,b^{4})\cos(x)^{4}+8\,a^{4}+8\,a^{3}b+3\,a^{2}b^{2}+2\,(8\,a^{3}b+8\,a^{2}b^{2}+3\,ab^{3})\cos(x)^{2}\right)\sqrt{a^{2}+ab}\,\arctan\left(\frac{(2\,a+b)\cos(x)^{2}-a}{2\,\sqrt{a^{2}+ab}\,\cos(x)\sin(x)}\right)+2\left(3\,(2\,a^{3}b^{2}+3\,a^{2}b^{3}+ab^{4})\cos(x)^{3}+(8\,a^{4}b+13\,a^{3}b^{2}+5\,a^{2}b^{3})\cos(x)\right)\sin(x)}{16\left(a^{8}+3\,a^{7}b+3\,a^{6}b^{2}+a^{5}b^{3}+(a^{6}b^{2}+3\,a^{5}b^{3}+3\,a^{4}b^{4}+a^{3}b^{5})\cos(x)^{4}+2\,(a^{7}b+3\,a^{6}b^{2}+3\,a^{5}b^{3}+a^{4}b^{4})\cos(x)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b
^2 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*cos(x)^2)*sqrt(-a^2 - a*b)*log(((8*a
```

```
^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b)*cos(x)^2 + 4*((2*a + b)*cos(
x)^3 - a*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2)/(b^2*cos(x)^4 + 2*a*b*cos(x
)^2 + a^2)) + 4*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 + (8*a^4*b + 13
*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b + 3*a^6*b^2 + a^5*b^3
+ (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 + 2*(a^7*b + 3*a^6*b
^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*
cos(x)^4 + 8*a^4 + 8*a^3*b + 3*a^2*b^2 + 2*(8*a^3*b + 8*a^2*b^2 + 3*a*b^3)*
cos(x)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a)/(sqrt(a^2 + a
*b)*cos(x)*sin(x))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^3 + (8*a^
4*b + 13*a^3*b^2 + 5*a^2*b^3)*cos(x))*sin(x))/(a^8 + 3*a^7*b + 3*a^6*b^2 +
a^5*b^3 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*cos(x)^4 + 2*(a^7*b +
 3*a^6*b^2 + 3*a^5*b^3 + a^4*b^4)*cos(x)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 149, normalized size = 1.39

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)}{\sqrt{a^2+ab}}\right)\right)(8\,a^2+8\,ab+3\,b^2)}{8\,(a^4+2\,a^3b+a^2b^2)\sqrt{a^2+ab}}-\frac{8\,a^2b\tan(x)^3+5\,ab^2\tan(x)^3+8\,a^2b\tan(x)+11\,ab^2\tan(x)+3\,b^3\tan(x)}{8\,(a^4+2\,a^3b+a^2b^2)\left(a\tan(x)^2+a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(pi*floor(x/pi + 1/2)*sgn(a) + arctan(a*tan(x)/sqrt(a^2 + a*b)))*(8*a^2
 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a^2 + a*b)) - 1/8*(8*a^2*
b*tan(x)^3 + 5*a*b^2*tan(x)^3 + 8*a^2*b*tan(x) + 11*a*b^2*tan(x) + 3*b^3*ta
n(x))/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(x)^2 + a + b)^2)
```

Mupad [**B**]
time = 2.44, size = 123, normalized size = 1.15

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a+b}}\right)\,\left(8\,a^2+8\,a\,b+3\,b^2\right)}{8\,a^{5/2}\,{\left(a+b\right)}^{5/2}}-\frac{\frac{\tan(x)\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,(a+b)}+\frac{\tan(x)^3\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,{(a+b)}^2}}{2\,a\,b+{\tan\left(x\right)}^2\,\left(2\,a^2+2\,b\,a\right)+a^2\,{\tan\left(x\right)}^4+a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^3,x)
```

```
[Out] (atan((a^(1/2)*tan(x))/(a + b)^(1/2))*(8*a*b + 8*a^2 + 3*b^2))/(8*a^(5/2)*(
a + b)^(5/2)) - ((tan(x)*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(x)^3*(8*a*
b + 5*b^2))/(8*a*(a + b)^2))/(2*a*b + tan(x)^2*(2*a*b + 2*a^2) + a^2*tan(x)
^4 + a^2 + b^2)
```

3.44 $\int \frac{1}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=34

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3260, 209}

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 15, normalized size = 0.44

$$\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-1),x]
[Out] ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.41

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{2}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.48, size = 13, normalized size = 0.38

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Fricas [A]
time = 0.43, size = 31, normalized size = 0.91

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A]
time = 0.23, size = 63, normalized size = 1.85

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/2 + sqrt(2
)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/2
```

Giac [A]
time = 0.39, size = 46, normalized size = 1.35

$$\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-\cos\left(2x\right)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1)))
```

Mupad [B]
time = 2.32, size = 26, normalized size = 0.76

$$\frac{\sqrt{2}\left(x-\operatorname{atan}(\tan(x))\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1),x)
```

```
[Out] (2^(1/2)*(x - atan(tan(x))))/2 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

3.45 $\int \frac{1}{(1+\cos^2(x))^2}\,dx$

Optimal. Leaf size=55

$$\frac{3x}{4\sqrt{2}}-\frac{3\mathrm{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\cos(x)\sin(x)}{4\left(1+\cos^2(x)\right)}$$

```
[Out] -1/4*cos(x)*sin(x)/(1+cos(x)^2)+3/8*x*2^(1/2)-3/8*arctan(cos(x)*sin(x)/(1+c
os(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3263, 12, 3260, 209}

$$-\frac{3\mathrm{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}+\frac{3x}{4\sqrt{2}}-\frac{\sin(x)\cos(x)}{4\left(\cos^2(x)+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-2),x]
```

```
[Out] (3*x)/(4*Sqrt[2]) - (3*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)])/(4
*Sqrt[2]) - (Cos[x]*Sin[x])/(4*(1 + Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
```

```
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cos^2(x))^2}\,dx &= -\frac{\cos(x)\sin(x)}{4(1+\cos^2(x))}-\frac{1}{4}\int -\frac{3}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{4(1+\cos^2(x))}+\frac{3}{4}\int \frac{1}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{4(1+\cos^2(x))}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\ &= \frac{3x}{4\sqrt{2}}-\frac{3\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\cos(x)\sin(x)}{4(1+\cos^2(x))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 35, normalized size = 0.64

$$\frac{3\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\sin(2x)}{4(3+\cos(2x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-2),x]
[Out] (3*ArcTan[Tan[x]/Sqrt[2]])/(4*Sqrt[2]) - Sin[2*x]/(4*(3 + Cos[2*x]))
```

Maple [A]
time = 0.06, size = 27, normalized size = 0.49

method	result	size
default	$-\frac{\tan(x)}{4(\tan^2(x)+2)}+\frac{3\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	27
risch	$-\frac{i(3\,e^{2ix}+1)}{2(e^{4ix}+6\,e^{2ix}+1)}+\frac{3i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{16}-\frac{3i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{16}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*tan(x)/(tan(x)^2+2)+3/8*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.47

$$\frac{3}{8}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(x\right)\right)-\frac{\tan\left(x\right)}{4\,\left(\tan\left(x\right)^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/4*tan(x)/(tan(x)^2 + 2)
```

Fricas [A]
time = 0.40, size = 57, normalized size = 1.04

$$-\frac{3\,\left(\sqrt{2}\,\cos\left(x\right)^{2}+\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^{2}-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,\cos\left(x\right)\sin\left(x\right)}{16\,\left(\cos\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(3*(sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt
(2))/(cos(x)*sin(x))) + 4*cos(x)*sin(x))/(cos(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(53) = 106$.
time = 1.09, size = 218, normalized size = 3.96

$$\frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^{4}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+8}+\frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{8\tan^{4}\left(\frac{x}{2}\right)+8}+\frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^{4}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+8}+\frac{3\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{8\tan^{4}\left(\frac{x}{2}\right)+8}+\frac{2\tan^{3}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+8}-\frac{2\tan\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**2,x)
```

```
[Out] 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)
**4/(8*tan(x/2)**4 + 8) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor(
(x/2 - pi/2)/pi))/(8*tan(x/2)**4 + 8) + 3*sqrt(2)*(atan(sqrt(2)*tan(x/2) +
1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(8*tan(x/2)**4 + 8) + 3*sqrt(2)
*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(8*tan(x/2)**4 +
8) + 2*tan(x/2)**3/(8*tan(x/2)**4 + 8) - 2*tan(x/2)/(8*tan(x/2)**4 + 8)
```

Giac [A]
time = 0.40, size = 59, normalized size = 1.07

$$\frac{3}{8}\,\sqrt{2}\,\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin\left(2\,x\right)-\sin\left(2\,x\right)}{\sqrt{2}\,\cos\left(2\,x\right)+\sqrt{2}\,-\cos\left(2\,x\right)+1}\right)\right)-\frac{\tan\left(x\right)}{4\,\left(\tan\left(x\right)^{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - 1/4*tan(x)/(tan(x)^2 + 2)
```

Mupad [B]
time = 2.17, size = 40, normalized size = 0.73

$$\frac{3\sqrt{2}\ (x-\mathrm{atan}(\tan(x)))}{8}-\frac{\tan(x)}{4\ \left(\tan\left(x\right)^2+2\right)}+\frac{3\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \tan(x)}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^2,x)
```

```
[Out] (3*2^(1/2)*(x - atan(tan(x))))/8 - tan(x)/(4*(tan(x)^2 + 2)) + (3*2^(1/2)*a
tan((2^(1/2)*tan(x))/2))/8
```

3.46 $\int \frac{1}{\left(1+\cos^2(x)\right)^3}\,dx$

Optimal. Leaf size=71

$$\frac{19x}{32\sqrt{2}}-\frac{19\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{32\sqrt{2}}-\frac{\cos(x)\sin(x)}{8\left(1+\cos^2(x)\right)^2}-\frac{9\cos(x)\sin(x)}{32\left(1+\cos^2(x)\right)}$$

```
[Out] -1/8*cos(x)*sin(x)/(1+cos(x)^2)^2-9/32*cos(x)*sin(x)/(1+cos(x)^2)+19/64*x*2
^(1/2)-19/64*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3263, 3252, 12, 3260, 209}

$$-\frac{19\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{32\sqrt{2}}+\frac{19x}{32\sqrt{2}}-\frac{9\sin(x)\cos(x)}{32\left(\cos^2(x)+1\right)}-\frac{\sin(x)\cos(x)}{8\left(\cos^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-3),x]
```

```
[Out] (19*x)/(32*Sqrt[2]) - (19*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)])
/(32*Sqrt[2]) - (Cos[x]*Sin[x])/(8*(1 + Cos[x]^2)^2) - (9*Cos[x]*Sin[x])/(3
2*(1 + Cos[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3252

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b - a*B))*Cos[e + f*x]*Sin[e + f*x
]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(a + b)*(p + 1))), x] - Dist[1/(2*
a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p
 + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cos^2(x))^3}\,dx &= -\frac{\cos(x)\sin(x)}{8(1+\cos^2(x))^2}-\frac{1}{8}\int \frac{-7+2\cos^2(x)}{(1+\cos^2(x))^2}\,dx\\ &= -\frac{\cos(x)\sin(x)}{8(1+\cos^2(x))^2}-\frac{9\cos(x)\sin(x)}{32(1+\cos^2(x))}-\frac{1}{32}\int -\frac{19}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{8(1+\cos^2(x))^2}-\frac{9\cos(x)\sin(x)}{32(1+\cos^2(x))}+\frac{19}{32}\int \frac{1}{1+\cos^2(x)}\,dx\\ &= -\frac{\cos(x)\sin(x)}{8(1+\cos^2(x))^2}-\frac{9\cos(x)\sin(x)}{32(1+\cos^2(x))}-\frac{19}{32}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(x)\right)\\ &= \frac{19x}{32\sqrt{2}}-\frac{19\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{32\sqrt{2}}-\frac{\cos(x)\sin(x)}{8(1+\cos^2(x))^2}-\frac{9\cos(x)\sin(x)}{32(1+\cos^2(x))}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 51, normalized size = 0.72

$$\frac{19\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{32\sqrt{2}}-\frac{\sin(2x)}{4(3+\cos(2x))^2}-\frac{9\sin(2x)}{32(3+\cos(2x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-3),x]
[Out] (19*ArcTan[Tan[x]/Sqrt[2]])/(32*Sqrt[2]) - Sin[2*x]/(4*(3 + Cos[2*x])^2) -
(9*Sin[2*x])/(32*(3 + Cos[2*x]))
```

Maple [A]
time = 0.07, size = 35, normalized size = 0.49

method	result	size
default	$\frac{-\frac{13\left(\tan^{3}\left(x\right)\right)}{32}-\frac{11\tan\left(x\right)}{16}}{\left(\tan^{2}\left(x\right)+2\right)^{2}}+\frac{19\arctan\left(\frac{\tan\left(x\right)\sqrt{2}}{2}\right)\sqrt{2}}{64}$	35
risch	$-\frac{i\left(19\,\mathrm{e}^{6ix}+171\,\mathrm{e}^{4ix}+89\,\mathrm{e}^{2ix}+9\right)}{16\left(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1\right)^{2}}+\frac{19i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{128}-\frac{19i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{128}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-13/32*tan(x)^3-11/16*tan(x))/(tan(x)^2+2)^2+19/64*arctan(1/2*tan(x)*2^(1/
2))*2^(1/2)
```

Maxima [A]
time = 0.47, size = 41, normalized size = 0.58

$$\frac{19}{64}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(x\right)\right)-\frac{13\,\tan\left(x\right)^{3}+22\,\tan\left(x\right)}{32\,\left(\tan\left(x\right)^{4}+4\,\tan\left(x\right)^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 19/64*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/32*(13*tan(x)^3 + 22*tan(x))/(
tan(x)^4 + 4*tan(x)^2 + 4)
```

Fricas [A]
time = 0.43, size = 81, normalized size = 1.14

$$-\frac{19\,\left(\sqrt{2}\,\cos\left(x\right)^{4}+2\,\sqrt{2}\,\cos\left(x\right)^{2}+\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(x\right)^{2}-\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)+4\,\left(9\,\cos\left(x\right)^{3}+13\,\cos\left(x\right)\right)\sin\left(x\right)}{128\,\left(\cos\left(x\right)^{4}+2\,\cos\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(19*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*
sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) + 4*(9*cos(x)^3 + 13*cos(x))*s
in(x))/(cos(x)^4 + 2*cos(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(71) = 142.
time = 3.58, size = 439, normalized size = 6.18

$$\frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^{8}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{38\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^{4}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^{8}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{38\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)\tan^{4}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{19\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{22\tan^{7}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}-\frac{14\tan^{5}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}+\frac{14\tan^{3}\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}-\frac{22\tan\left(\frac{x}{2}\right)}{64\tan^{8}\left(\frac{x}{2}\right)+128\tan^{4}\left(\frac{x}{2}\right)+64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**3,x)
```

```
[Out] 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2
)**8/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 38*sqrt(2)*(atan(sqrt(2)*tan
(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**4/(64*tan(x/2)**8 + 128*t
an(x/2)**4 + 64) + 19*sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 -
 pi/2)/pi))/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 19*sqrt(2)*(atan(sqrt
(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**8/(64*tan(x/2)**8
+ 128*tan(x/2)**4 + 64) + 38*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor
((x/2 - pi/2)/pi))*tan(x/2)**4/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 19
*sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(64*tan(x
/2)**8 + 128*tan(x/2)**4 + 64) + 22*tan(x/2)**7/(64*tan(x/2)**8 + 128*tan(x
/2)**4 + 64) - 14*tan(x/2)**5/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) + 14*
tan(x/2)**3/(64*tan(x/2)**8 + 128*tan(x/2)**4 + 64) - 22*tan(x/2)/(64*tan(x
/2)**8 + 128*tan(x/2)**4 + 64)
```

Giac [A]
time = 0.40, size = 68, normalized size = 0.96

$$\frac{19}{64}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2\,x\right)-\sin\left(2\,x\right)}{\sqrt{2}\cos\left(2\,x\right)+\sqrt{2}-\cos\left(2\,x\right)+1}\right)\right)-\frac{13\,\tan\left(x\right)^{3}+22\,\tan\left(x\right)}{32\,\left(\tan\left(x\right)^{2}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^3,x, algorithm="giac")
```

```
[Out] 19/64*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - cos(2*x) + 1))) - 1/32*(13*tan(x)^3 + 22*tan(x))/(tan(x)^2 + 2)
^2
```

Mupad [B]
time = 2.15, size = 53, normalized size = 0.75

$$\frac{19\,\sqrt{2}\,\left(x-\mathrm{atan}(\tan(x))\right)}{64}-\frac{\frac{13\tan(x)^3}{32}+\frac{11\tan(x)}{16}}{\tan\left(x\right)^4+4\tan\left(x\right)^2+4}+\frac{19\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^3,x)
```

```
[Out] (19*2^(1/2)*(x - atan(tan(x))))/64 - ((11*tan(x))/16 + (13*tan(x)^3)/32)/(4
*tan(x)^2 + tan(x)^4 + 4) + (19*2^(1/2)*atan((2^(1/2)*tan(x))/2))/64
```

3.47 $\int \sqrt{1-\cos^2(x)}\,dx$

Optimal. Leaf size=12

$$-\cot(x)\sqrt{\sin^2(x)}$$

```
[Out] -cot(x)*(sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3255, 3286, 2718}

$$\sqrt{\sin^2(x)}\,(-\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[Sin[x]^2])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\cos^2(x)}\,dx &= \int \sqrt{\sin^2(x)}\,dx\\ &= \left(\csc(x)\sqrt{\sin^2(x)}\right)\int \sin(x)\,dx\\ &= -\cot(x)\sqrt{\sin^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\cot(x)\sqrt{\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]^2],x]
[Out] -(Cot[x]*Sqrt[Sin[x]^2])
```

Maple [A]
time = 0.24, size = 13, normalized size = 1.08

method	result	size
default	$-\frac{2\sin(x)\cos(x)}{\sqrt{2-2\cos(2x)}}$	13
risch	$-\frac{i\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}\, e^{2ix}}{2\left(e^{2ix}-1\right)}-\frac{i\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}{2\left(e^{2ix}-1\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -sin(x)*cos(x)/(sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(tan(x)^2 + 1)
```

Fricas [A]
time = 0.40, size = 4, normalized size = 0.33

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\cos^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.
time = 0.40, size = 24, normalized size = 2.00

$$-\frac{2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sgn(tan(1/2*x)^3 + tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.83

$$-\cot(x)\,\sqrt{\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x)^2)^(1/2),x)
```

```
[Out] -cot(x)*(sin(x)^2)^(1/2)
```

3.48 $\int \sqrt{-1+\cos^2(x)}\,dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{-\sin^2(x)}$$

```
[Out] -cot(x)*(-sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3255, 3286, 2718}

$$\sqrt{-\sin^2(x)}\,(-\cot(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Cos[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[-Sin[x]^2])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+\cos^2(x)}\,dx &= \int \sqrt{-\sin^2(x)}\,dx\\ &= \left(\csc(x)\sqrt{-\sin^2(x)}\right)\int \sin(x)\,dx\\ &= -\cot(x)\sqrt{-\sin^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\cot(x)\sqrt{-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Cos[x]^2],x]
[Out] -(Cot[x]*Sqrt[-Sin[x]^2])
```

Maple [A]
time = 0.30, size = 14, normalized size = 1.00

method	result	size
default	$\frac{\sin(x)\cos(x)}{\sqrt{-\left(\sin^2(x)\right)}}$	14
risch	$-\frac{i\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}\,e^{2ix}}{2(e^{2ix}-1)}-\frac{i\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}{2(e^{2ix}-1)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] sin(x)*cos(x)/(-sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 12, normalized size = 0.86

$$-\frac{1}{\sqrt{-\tan\left(x\right)^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/sqrt(-tan(x)^2 - 1)
```

Fricas [F]
time = 0.38, size = 1, normalized size = 0.07

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)**2)**(1/2),x)
[Out] Integral(sqrt(cos(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 28, normalized size = 2.00

$$\frac{2i\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3 - \tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*I*sgn(-tan(1/2*x)^3 - tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 2.29, size = 39, normalized size = 2.79

$$-\frac{\sqrt{-4\sin(x)^2}\,\left(-\sin(x)^2 + \frac{\sin(2\,x)\,1\mathrm{i}}{2} + 1\right)}{\sin(x)^2\,2\mathrm{i} + \sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 - 1)^(1/2),x)
[Out] -((-4*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(sin(2*x) + sin(x)^
2*2i)
```

3.49 $\int (1-\cos^2(x))^{3/2} \, dx$

Optimal. Leaf size=29

$$-\frac{2}{3}\cot(x)\sqrt{\sin^2(x)}-\frac{1}{3}\cot(x)\sin^2(x)^{3/2}$$

```
[Out] -1/3*cot(x)*(sin(x)^2)^(3/2)-2/3*cot(x)*(sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3255, 3282, 3286, 2718}

$$-\frac{1}{3}\sin^2(x)^{3/2}\cot(x)-\frac{2}{3}\sqrt{\sin^2(x)}\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(3/2),x]
```

```
[Out] (-2*Cot[x]*Sqrt[Sin[x]^2])/3 - (Cot[x]*(Sin[x]^2)^(3/2))/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(1-\cos^2(x)\right)^{3/2}\,dx &= \int \sin^2(x)^{3/2}\,dx\\ &= -\frac{1}{3}\cot(x)\sin^2(x)^{3/2}+\frac{2}{3}\int \sqrt{\sin^2(x)}\,dx\\ &= -\frac{1}{3}\cot(x)\sin^2(x)^{3/2}+\frac{1}{3}\left(2\csc(x)\sqrt{\sin^2(x)}\right)\int \sin(x)\,dx\\ &= -\frac{2}{3}\cot(x)\sqrt{\sin^2(x)}-\frac{1}{3}\cot(x)\sin^2(x)^{3/2}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 23, normalized size = 0.79

$$\frac{1}{12}(-9\cos(x)+\cos(3x))\csc(x)\sqrt{\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(3/2),x]
[Out] ((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[Sin[x]^2])/12
```

Maple [**A**]
time = 0.24, size = 19, normalized size = 0.66

method	result
default	$\frac{2\sin(x)\cos(x)\left(\cos^2(x)-3\right)}{3\sqrt{2-2\cos(2x)}}$
risch	$\frac{i e^{4ix}\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}{24\,e^{2ix}-24}-\frac{3i\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}\,e^{2ix}}{8(e^{2ix}-1)}-\frac{3i\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}{8(e^{2ix}-1)}+\frac{i e^{-2ix}\sqrt{-(e^{2}}}{24\,e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*sin(x)*cos(x)*(cos(x)^2-3)/(sin(x)^2)^(1/2)
```

Maxima [**A**]
time = 0.52, size = 11, normalized size = 0.38

$$-\frac{1}{12}\cos\left(3\,x\right)+\frac{3}{4}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*cos(3*x) + 3/4*cos(x)
```

Fricas [A]
time = 0.39, size = 11, normalized size = 0.38

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(1-\cos^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - cos(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.
time = 0.41, size = 45, normalized size = 1.55

$$-\frac{4\left(3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^2+\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -4/3*(3*sgn(tan(1/2*x)^3 + tan(1/2*x))*tan(1/2*x)^2 + sgn(tan(1/2*x)^3 + ta
n(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int{\left(1-{\cos\left(x\right)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x)^2)^(3/2),x)
```

```
[Out] int((1 - cos(x)^2)^(3/2), x)
```

3.50 $\int \left(-1+\cos^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=33

$$\frac{2}{3}\cot(x)\sqrt{-\sin^2(x)} - \frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2}$$

```
[Out] -1/3*cot(x)*(-sin(x)^2)^(3/2)+2/3*cot(x)*(-sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3255, 3282, 3286, 2718}

$$\frac{2}{3}\sqrt{-\sin^2(x)}\cot(x) - \frac{1}{3}\left(-\sin^2(x)\right)^{3/2}\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Cos[x]^2)^(3/2),x]
```

```
[Out] (2*Cot[x]*Sqrt[-Sin[x]^2])/3 - (Cot[x]*(-Sin[x]^2)^(3/2))/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \left(-1+\cos^2(x)\right)^{3/2}\,dx &= \int \left(-\sin^2(x)\right)^{3/2}\,dx\\
&= -\frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2} - \frac{2}{3}\int \sqrt{-\sin^2(x)}\,dx\\
&= -\frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2} - \frac{1}{3}\left(2\csc(x)\sqrt{-\sin^2(x)}\right)\int \sin(x)\,dx\\
&= \frac{2}{3}\cot(x)\sqrt{-\sin^2(x)} - \frac{1}{3}\cot(x)\left(-\sin^2(x)\right)^{3/2}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.76

$$
-\frac{1}{12}(-9\cos(x)+\cos(3x))\csc(x)\sqrt{-\sin^2(x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cos[x]^2)^(3/2),x]
[Out] -1/12*((-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[-Sin[x]^2])
```

Maple [A]
time = 0.28, size = 21, normalized size = 0.64

method	result
default	$-\frac{\sin(x)\cos(x)(\sin^2(x)+2)}{3\sqrt{-\left(\sin^2(x)\right)}}$
risch	$-\frac{i e^{4ix}\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}{24(e^{2ix}-1)} + \frac{3i\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}\,e^{2ix}}{8(e^{2ix}-1)} + \frac{3i\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}{8(e^{2ix}-1)} - \frac{i e^{-2ix}\sqrt{\left(e^{2ix}-1\right)^2}}{24(e^{2ix}-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3*sin(x)*cos(x)*(sin(x)^2+2)/(-sin(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((cos(x)^2 - 1)^(3/2), x)
```

Fricas [F]
time = 0.39, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] 0
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(x\right) - 1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)**2)**(3/2),x)
[Out] Integral((cos(x)**2 - 1)**(3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 55, normalized size = 1.67

$$-\frac{4\left(3i\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{3}-\tan\left(\frac{1}{2}\,x\right)\right)\tan\left(\frac{1}{2}\,x\right)^{2}+i\operatorname{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^{3}-\tan\left(\frac{1}{2}\,x\right)\right)\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+cos(x)^2)^(3/2),x, algorithm="giac")
[Out] -4/3*(3*I*sgn(-tan(1/2*x)^3 - tan(1/2*x))*tan(1/2*x)^2 + I*sgn(-tan(1/2*x)^
3 - tan(1/2*x)))/(tan(1/2*x)^2 + 1)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int {\left({\cos\left(x\right)}^2-1\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 - 1)^(3/2),x)
[Out] int((cos(x)^2 - 1)^(3/2), x)
```

3.51 $\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx$

Optimal. Leaf size=15

$$-\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{\sin^2(x)}}$$

```
[Out] -arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3255, 3286, 3855}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cos[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[Sin[x]^2])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{\sin^2(x)}}\,dx$$
$$= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{\sin^2(x)}}$$
$$= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{\sin^2(x)}}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.87

$$\frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cos[x]^2],x]
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[Sin[x]^2]
```

Maple [A]
time = 0.24, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{2\,\mathrm{arctanh}(\cos(x))\sin(x)}{\sqrt{2-2\cos(2x)}}$	14
risch	$\frac{2\ln(\mathrm{e}^{ix}-1)\sin(x)}{\sqrt{-\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}-\frac{2\ln(\mathrm{e}^{ix}+1)\sin(x)}{\sqrt{-\left(\mathrm{e}^{2ix}-1\right)^2\mathrm{e}^{-2ix}}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.
time = 0.51, size = 35, normalized size = 2.33

$$\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)-\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*log(cos(x)^2 + sin(x)^2 -
 2*cos(x) + 1)
```

Fricas [A]
time = 0.40, size = 19, normalized size = 1.27

$$-\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\cos^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cos(x)**2), x)
```

Giac [A]
time = 0.41, size = 21, normalized size = 1.40

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{3}+\tan\left(\frac{1}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/sgn(tan(1/2*x)^3 + tan(1/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{1-{\cos\left(x\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cos(x)^2)^(1/2),x)
```

```
[Out] int(1/(1 - cos(x)^2)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{-\sin^2(x)}}$$

```
[Out] -arctanh(cos(x))*sin(x)/(-sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3255, 3286, 3855}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Cos[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[-Sin[x]^2])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{-\sin^2(x)}}\,dx$$
$$= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{-\sin^2(x)}}$$
$$= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{-\sin^2(x)}}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.76

$$\frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Cos[x]^2],x]
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[-Sin[x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.21, size = 34, normalized size = 2.00

method	result	size
default	$-\frac{\sin(x)\sqrt{-\left(\cos^2(x)\right)}\arctan\left(\frac{1}{\sqrt{-\left(\cos^2(x)\right)}}\right)}{\cos(x)\sqrt{-\left(\sin^2(x)\right)}}$	34
risch	$-\frac{2\ln(e^{ix}+1)\sin(x)}{\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}+\frac{2\ln(e^{ix}-1)\sin(x)}{\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -sin(x)*(-cos(x)^2)^(1/2)*arctan(1/(-cos(x)^2)^(1/2))/cos(x)/(-sin(x)^2)^(1
/2)
```

Maxima [A]
time = 0.53, size = 17, normalized size = 1.00

$$-\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)+\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan2(sin(x), cos(x) + 1) + arctan2(sin(x), cos(x) - 1)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(x)**2 - 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 27, normalized size = 1.59

$$\frac{i\log\left(-i\tan\left(\frac{1}{2}x\right)\right)}{\operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] I*log(-I*tan(1/2*x))/sgn(-tan(1/2*x)^3 - tan(1/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{\cos(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(cos(x)^2 - 1)^(1/2), x)
```

3.53 $\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}-\frac{\tanh^{-1}(\cos(x))\sin(x)}{2\sqrt{\sin^2(x)}}$$

```
[Out] -1/2*cot(x)/(sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/(sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3255, 3283, 3286, 3855}

$$-\frac{\cot(x)}{2\sqrt{\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)^(-3/2),x]
```

```
[Out] -1/2*Cot[x]/Sqrt[Sin[x]^2] - (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[Sin[x]^2])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-\cos^2(x)\right)^{3/2}}\,dx &= \int \frac{1}{\sin^2(x)^{3/2}}\,dx\\
&= -\frac{\cot(x)}{2\sqrt{\sin^2(x)}}+\frac{1}{2}\int \frac{1}{\sqrt{\sin^2(x)}}\,dx\\
&= -\frac{\cot(x)}{2\sqrt{\sin^2(x)}}+\frac{\sin(x)\int \csc(x)\,dx}{2\sqrt{\sin^2(x)}}\\
&= -\frac{\cot(x)}{2\sqrt{\sin^2(x)}}-\frac{\tanh^{-1}(\cos(x))\sin(x)}{2\sqrt{\sin^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 1.59

$$
-\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin(x)}{8\sqrt{\sin^2(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)^(-3/2),x]
[Out] -1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x])
/Sqrt[Sin[x]^2]
```

Maple [A]
time = 0.35, size = 37, normalized size = 1.16

method	result	size
default	$-\frac{2\left(\frac{\cos(x)}{2}+\frac{(\ln(\cos(x)+1)-\ln(-1+\cos(x)))\left(\sin^2(x)\right)}{4}\right)}{\sin(x)\sqrt{2-2\cos(2x)}}$	37
risch	$-\frac{i\left(e^{2ix}+1\right)}{\left(e^{2ix}-1\right)\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}+\frac{\ln\left(e^{ix}-1\right)\sin(x)}{\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}-\frac{\ln\left(e^{ix}+1\right)\sin(x)}{\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(1/2*cos(x)+1/4*(ln(cos(x)+1)-ln(-1+cos(x)))*sin(x)^2)/sin(x)/(sin(x)^2)^(
1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(24) = 48$.
time = 0.52, size = 300, normalized size = 9.38

$$\frac{4\left(\cos\left(3x\right)+\cos\left(x\right)\right)\cos\left(4x\right)-4\left(2\cos\left(2x\right)-1\right)\cos\left(3x\right)-8\cos\left(2x\right)\cos\left(x\right)+\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\left(2x\right)-1\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right)-\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\left(2x\right)-1\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)+4\left(\sin\left(3x\right)+\sin\left(x\right)\right)\sin\left(4x\right)-8\sin\left(3x\right)\sin\left(2x\right)-8\sin\left(2x\right)\sin\left(x\right)+4\cos\left(x\right)}{4\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\left(2x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*x) + cos(x))*cos(4*x) - 4*(2*cos(2*x) - 1)*cos(3*x) - 8*cos(2
*x)*cos(x) + (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin
(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*log(cos(x)^2
 + sin(x)^2 + 2*cos(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4
*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x)
- 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*(sin(3*x) + sin(x))*sin(4*
x) - 8*sin(3*x)*sin(2*x) - 8*sin(2*x)*sin(x) + 4*cos(x))/(2*(2*cos(2*x) - 1
)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) -
 4*sin(2*x)^2 + 4*cos(2*x) - 1)
```

Fricas [A]
time = 0.38, size = 44, normalized size = 1.38

$$-\frac{\left(\cos\left(x\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-\left(\cos\left(x\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)-2\cos\left(x\right)}{4\left(\cos\left(x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(1-\cos^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((1 - cos(x)**2)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(24) = 48$.
time = 0.42, size = 78, normalized size = 2.44

$$\frac{\tan\left(\frac{1}{2}x\right)^2}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)}+\frac{\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{4\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)}-\frac{2\,\tan\left(\frac{1}{2}x\right)^2+1}{8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*tan(1/2*x)^2/sgn(tan(1/2*x)^3 + tan(1/2*x)) + 1/4*log(tan(1/2*x)^2)/sgn
(tan(1/2*x)^3 + tan(1/2*x)) - 1/8*(2*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x)^3 +
tan(1/2*x))*tan(1/2*x)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(1-\cos\left(x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(1 - cos(x)^2)^(3/2), x)
```

3.54 $\int \frac{1}{\left(-1+\cos^2(x)\right)^{3/2}} dx$

Optimal. Leaf size=36

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}+\frac{\tanh^{-1}(\cos(x))\sin(x)}{2\sqrt{-\sin^2(x)}}$$

```
[Out] 1/2*cot(x)/(-sin(x)^2)^(1/2)+1/2*arctanh(cos(x))*sin(x)/(-sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3255, 3283, 3286, 3855}

$$\frac{\cot(x)}{2\sqrt{-\sin^2(x)}}+\frac{\sin(x)\tanh^{-1}(\cos(x))}{2\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] Cot[x]/(2*Sqrt[-Sin[x]^2]) + (ArcTanh[Cos[x]]*Sin[x])/(2*Sqrt[-Sin[x]^2])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+\cos^2(x))^{3/2}}\,dx &= \int \frac{1}{(-\sin^2(x))^{3/2}}\,dx\\
&= \frac{\cot(x)}{2\sqrt{-\sin^2(x)}}-\frac{1}{2}\int \frac{1}{\sqrt{-\sin^2(x)}}\,dx\\
&= \frac{\cot(x)}{2\sqrt{-\sin^2(x)}}-\frac{\sin(x)\int \csc(x)\,dx}{2\sqrt{-\sin^2(x)}}\\
&= \frac{\cot(x)}{2\sqrt{-\sin^2(x)}}+\frac{\tanh^{-1}(\cos(x))\sin(x)}{2\sqrt{-\sin^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.47

$$\frac{\left(\csc^2\left(\frac{x}{2}\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)-\sec^2\left(\frac{x}{2}\right)\right)\sin(x)}{8\sqrt{-\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Cos[x]^2)^(-3/2),x]
[Out] ((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x])/(8*S
qrt[-Sin[x]^2])
```

Maple [A]
time = 0.31, size = 52, normalized size = 1.44

method	result	size
default	$\frac{\sqrt{-\left(\cos^2\left(x\right)\right)}\left(\arctan\left(\frac{1}{\sqrt{-\left(\cos^2\left(x\right)\right)}}\right)\left(\sin^2(x)\right)-\sqrt{-\left(\cos^2\left(x\right)\right)}\right)}{2\sin(x)\cos(x)\sqrt{-\left(\sin^2\left(x\right)\right)}}$	52
risch	$\frac{i\left(e^{2ix}+1\right)}{\left(e^{2ix}-1\right)\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}-\frac{\ln\left(e^{ix}-1\right)\sin(x)}{\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}+\frac{\ln\left(e^{ix}+1\right)\sin(x)}{\sqrt{\left(e^{2ix}-1\right)^2 e^{-2ix}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-cos(x)^2)^(1/2)*(arctan(1/(-cos(x)^2)^(1/2))*sin(x)^2-(-cos(x)^2)^(1/
2))/sin(x)/cos(x)/(-sin(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(28) = 56$.
time = 0.51, size = 284, normalized size = 7.89

$$\frac{(2\,(2\,\cos(2\,x)-1)\cos(4\,x)-\cos(4\,x)^2-4\,\cos(2\,x)^2-\sin(4\,x)^2+4\,\sin(4\,x)\sin(2\,x)-4\,\sin(2\,x)^2+4\,\cos(2\,x)-1)\arctan(\sin(x)\,,\cos(x)+1)-(2\,(2\,\cos(2\,x)-1)\cos(4\,x)-\cos(4\,x)^2-4\,\cos(2\,x)^2-\sin(4\,x)^2+4\,\sin(4\,x)\sin(2\,x)-4\,\sin(2\,x)^2+4\,\cos(2\,x)-1)\arctan(\sin(x)\,,\cos(x)-1)+2\,(\sin(3\,x)+\sin(x))\cos(4\,x)-2\,(\cos(3\,x)+\cos(x))\sin(4\,x)-2\,(2\,\cos(2\,x)-1)\sin(3\,x)+4\,\cos(3\,x)\sin(2\,x)+4\,\cos(x)\sin(2\,x)-4\,\cos(2\,x)\sin(x)+2\,\sin(x)}{2\,(2\,(2\,\cos(2\,x)-1)\cos(4\,x)-\cos(4\,x)^2-4\,\cos(2\,x)^2-\sin(4\,x)^2+4\,\sin(4\,x)\sin(2\,x)-4\,\sin(2\,x)^2+4\,\cos(2\,x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
+ 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos(
x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*
x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x),
 cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(4
*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x)
 - 4*cos(2*x)*sin(x) + 2*sin(x))/(2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2
- 4*cos(2*x)^2 - sin(4*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*
x) - 1)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos^2(x)-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 - 1)**(-3/2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 90, normalized size = 2.50

$$-\frac{i\tan\left(\frac{1}{2}x\right)^{2}}{8\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^{3}-\tan\left(\frac{1}{2}x\right)\right)}-\frac{i\log\left(\tan\left(\frac{1}{2}x\right)^{2}\right)}{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^{3}-\tan\left(\frac{1}{2}x\right)\right)}+\frac{2i\tan\left(\frac{1}{2}x\right)^{2}+i}{8\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^{3}-\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*I*tan(1/2*x)^2/sgn(-tan(1/2*x)^3 - tan(1/2*x)) - 1/4*I*log(tan(1/2*x)^
2)/sgn(-tan(1/2*x)^3 - tan(1/2*x)) + 1/8*(2*I*tan(1/2*x)^2 + I)/(sgn(-tan(1
/2*x)^3 - tan(1/2*x))*tan(1/2*x)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(\cos\left(x\right)^{2}-1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int(1/(cos(x)^2 - 1)^(3/2), x)
```

3.55 $\int \sqrt{1+\cos^2(x)}\,dx$

Optimal. Leaf size=9

$$E\left(\frac{\pi}{2}+x\middle|-1\right)$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3256}

$$E\left(x+\frac{\pi}{2}\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Cos[x]^2],x]
```

```
[Out] EllipticE[Pi/2 + x, -1]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{1+\cos^2(x)}\,dx = E\left(\frac{\pi}{2}+x\middle|-1\right)$$

Mathematica [A]
time = 0.03, size = 11, normalized size = 1.22

$$\sqrt{2}\,E\left(x\middle|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[2]*EllipticE[x, 1/2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.
time = 0.32, size = 41, normalized size = 4.56

method	result	size
default	$-\frac{\sqrt{\left(1+\cos^{2}\left(x\right)\right)\left(\sin^{2}\left(x\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos\left(2x\right)}{2}}\,\operatorname{EllipticE}\left(\cos\left(x\right),i\right)}{\sqrt{1-\left(\cos^{4}\left(x\right)\right)}\,\sin\left(x\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)*EllipticE(cos(x),I)/(1-cos(
x)^4)^(1/2)/sin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(cos(x)^2 + 1), x)
```

Fricas [F]
time = 0.08, size = 10, normalized size = 1.11

$$\operatorname{integral}\left(\sqrt{\cos\left(x\right)^{2}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(cos(x)^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\cos^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)**(1/2),x)
[Out] Integral(sqrt(cos(x)**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(x)^2 + 1), x)
```

Mupad [B]
time = 0.01, size = 7, normalized size = 0.78

$$\sqrt{2}\,\mathrm{E}\left(x\,\middle|\,\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*ellipticE(x, 1/2)
```

3.56 $\int \sqrt{-1-\cos^2(x)}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^
2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3257, 3256}

$$\frac{\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] (Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\cos^2(x)}\,dx &= \frac{\sqrt{-1-\cos^2(x)}\int \sqrt{1+\cos^2(x)}\,dx}{\sqrt{1+\cos^2(x)}}\\ &= \frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 1.06

$$-\frac{\sqrt{2}\,\sqrt{3+\cos(2x)}\,E\left(x\middle|\frac{1}{2}\right)}{\sqrt{-3-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] -((Sqrt[2]*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2])/Sqrt[-3 - Cos[2*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(35) = 70.
time = 0.39, size = 75, normalized size = 2.34

method	result	s
default	$-\frac{i\sqrt{-\left(1+\cos^{2}\left(x\right)\right)\left(\sin^{2}\left(x\right)\right)}\,\sqrt{1+\cos^{2}\left(x\right)}\,\sqrt{\frac{1}{2}-\frac{\cos\left(2x\right)}{2}}\,\left(2\,\mathrm{EllipticF}\left(i\cos\left(x\right),i\right)-\mathrm{EllipticE}\left(i\cos\left(x\right),i\right)\right)}{\sqrt{\cos^{4}\left(x\right)-1}\,\sin\left(x\right)\sqrt{-1-\left(\cos^{2}\left(x\right)\right)}}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*(sin(x)^2)^(1/2)*(2*El
lipticF(I*cos(x),I)-EllipticE(I*cos(x),I))/(cos(x)^4-1)^(1/2)/sin(x)/(-1-co
s(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(x)^2 - 1), x)
```

Fricas [F]
time = 0.08, size = 112, normalized size = 3.50

$$\frac{2\left(e^{(2ix)}-e^{(ix)}\right)\mathrm{integral}\left(\frac{4\sqrt{e^{(4ix)}+6\,e^{(2ix)}+1}\,\left(e^{(2ix)}+1\right)}{e^{(6ix)}-2\,e^{(5ix)}+7\,e^{(4ix)}-12\,e^{(3ix)}+7\,e^{(2ix)}-2\,e^{(ix)}+1},x\right)+\sqrt{e^{(4ix)}+6\,e^{(2ix)}+1}\,\left(e^{(ix)}+1\right)}{2\left(e^{(2ix)}-e^{(ix)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(e^(2*I*x) - e^(I*x))*integral(4*sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(
e^(2*I*x) + 1)/(e^(6*I*x) - 2*e^(5*I*x) + 7*e^(4*I*x) - 12*e^(3*I*x) + 7*e^
(2*I*x) - 2*e^(I*x) + 1), x) + sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(e^(I*x) +
 1))/(e^(2*I*x) - e^(I*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(x)**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(x)^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{-\cos(x)^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cos(x)^2 - 1)^(1/2),x)
```

```
[Out] int((- cos(x)^2 - 1)^(1/2), x)
```

3.57 $\int \sqrt{a+b\cos^2(x)}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{\sqrt{1+\frac{b\cos^2(x)}{a}}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2
)/(1+b*cos(x)^2/a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3257, 3256}

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/Sqrt[1 + (b*Cos[x]^2)/a]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cos^2(x)}\,dx &= \frac{\sqrt{a+b\cos^2(x)}\int \sqrt{1+\frac{b\cos^2(x)}{a}}\,dx}{\sqrt{1+\frac{b\cos^2(x)}{a}}}\\ &= \frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{\sqrt{1+\frac{b\cos^2(x)}{a}}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 1.10

$$\frac{\sqrt{2a+b+b\cos(2x)}\,E\left(x\middle|\frac{b}{a+b}\right)}{\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[2*a + b + b*Cos[2*x]]*EllipticE[x, b/(a + b)])/Sqrt[(2*a + b + b*Cos[
2*x])/(a + b)]
```

Maple [A]
time = 0.26, size = 49, normalized size = 1.17

method	result	size
default	$-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b(\cos^2(x))}{a}}\,\mathrm{EllipticE}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b\left(\cos^2\left(x\right)\right)}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(cos(x),(-1/a*b)^(1/2
))/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(x)^2 + a), x)
```

Fricas [F]
time = 0.10, size = 12, normalized size = 0.29

$$\text{integral}\left(\sqrt{b\cos\left(x\right)^2+a},\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(x)^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos^2{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\cos(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)^(1/2),x)
```

```
[Out] int((a + b*cos(x)^2)^(1/2), x)
```

3.58 $\int \left(1+\cos^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=43

$$2E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{2}{3}F\left(\frac{\pi}{2}+x\middle|-1\right)+\frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\,\sin(x)$$

```
[Out] -2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)+2/3*(sin(x)^2)^(1/2)/sin(x)*
EllipticF(cos(x),I)+1/3*cos(x)*sin(x)*(1+cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3259, 3251, 3256, 3261}

$$-\frac{2}{3}F\left(x+\frac{\pi}{2}\middle|-1\right)+2E\left(x+\frac{\pi}{2}\middle|-1\right)+\frac{1}{3}\sin(x)\cos(x)\sqrt{\cos^2(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(3/2),x]
```

```
[Out] 2*EllipticE[Pi/2 + x, -1] - (2*EllipticF[Pi/2 + x, -1])/3 + (Cos[x]*Sqrt[1
+ Cos[x]^2]*Sin[x])/3
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
```

```
0]
```

Rubi steps

$$\int \left(1+\cos^2(x)\right)^{3/2} dx = \frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\sin(x)+\frac{1}{3}\int \frac{4+6\cos^2(x)}{\sqrt{1+\cos^2(x)}}dx$$
$$= \frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\sin(x)-\frac{2}{3}\int \frac{1}{\sqrt{1+\cos^2(x)}}dx+2\int \sqrt{1+\cos^2(x)}\,dx$$
$$= 2E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{2}{3}F\left(\frac{\pi}{2}+x\middle|-1\right)+\frac{1}{3}\cos(x)\sqrt{1+\cos^2(x)}\sin(x)$$

Mathematica [A]
time = 0.06, size = 39, normalized size = 0.91

$$\frac{24E\left(x\middle|\frac{1}{2}\right)-4F\left(x\middle|\frac{1}{2}\right)+\sqrt{3+\cos(2x)}\sin(2x)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(3/2),x]
[Out] (24*EllipticE[x, 1/2] - 4*EllipticF[x, 1/2] + Sqrt[3 + Cos[2*x]]*Sin[2*x])/
(6*Sqrt[2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.
time = 0.49, size = 101, normalized size = 2.35

method	result
default	$\frac{\sqrt{(1+\cos^2(x))(\sin^2(x))}\left(-\cos(x)(\sin^4(x))+2\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{-(\sin^2(x))+2}\,\text{EllipticF}(\cos(x),i)-6\sqrt{\frac{1}{2}\right.}{3\sqrt{1-(\cos^4(x))}\sin(x)\sqrt{1+\cos^2(x)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*((1+cos(x)^2)*sin(x)^2)^(1/2)*(-cos(x)*sin(x)^4+2*(sin(x)^2)^(1/2)*(-si
n(x)^2+2)^(1/2)*EllipticF(cos(x),I)-6*(sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*
EllipticE(cos(x),I)+2*sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)^
2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x)^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.09, size = 10, normalized size = 0.23

$$\text{integral}\left(\left(\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(x)^2 + 1)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(x\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x)^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left({\cos\left(x\right)}^2+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)^(3/2),x)
```

```
[Out] int((cos(x)^2 + 1)^(3/2), x)
```

3.59 $\int \left(-1-\cos^2(x)\right)^{3/2} dx$

Optimal. Leaf size=89

$$-\frac{2\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}-\frac{2\sqrt{1+\cos^2(x)}\,F\left(\frac{\pi}{2}+x\middle|-1\right)}{3\sqrt{-1-\cos^2(x)}}-\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)$$

```
[Out] -1/3*cos(x)*sin(x)*(-1-cos(x)^2)^(1/2)+2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(
cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^2)^(1/2)+2/3*(sin(x)^2)^(1/2)/sin(x
)*EllipticF(cos(x),I)*(1+cos(x)^2)^(1/2)/(-1-cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$-\frac{1}{3}\sin(x)\cos(x)\sqrt{-\cos^2(x)-1}-\frac{2\sqrt{\cos^2(x)+1}\,F\left(x+\frac{\pi}{2}\middle|-1\right)}{3\sqrt{-\cos^2(x)-1}}-\frac{2\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cos[x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/Sqrt[1 + Cos[x]^2] - (2*Sq
rt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/(3*Sqrt[-1 - Cos[x]^2]) - (Cos[x]
*Sqrt[-1 - Cos[x]^2]*Sin[x])/3
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1-\cos^2(x)\right)^{3/2}\,dx &= -\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)+\frac{1}{3}\int\frac{4+6\cos^2(x)}{\sqrt{-1-\cos^2(x)}}\,dx\\ &= -\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)-\frac{2}{3}\int\frac{1}{\sqrt{-1-\cos^2(x)}}\,dx-2\int\sqrt{-1-\cos\\ &= -\frac{1}{3}\cos(x)\sqrt{-1-\cos^2(x)}\,\sin(x)-\frac{\left(2\sqrt{-1-\cos^2(x)}\right)\int\sqrt{1+\cos^2(x)}\,dx}{\sqrt{1+\cos^2(x)}}-\\ &= -\frac{2\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{1+\cos^2(x)}}-\frac{2\sqrt{1+\cos^2(x)}\,F\left(\frac{\pi}{2}+x\middle|-1\right)}{3\sqrt{-1-\cos^2(x)}}-\frac{1}{3}\cos(x\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.74

$$\frac{48\sqrt{3+\cos(2x)}\,E\left(x\middle|\frac{1}{2}\right)-8\sqrt{3+\cos(2x)}\,F\left(x\middle|\frac{1}{2}\right)+6\sin(2x)+\sin(4x)}{12\sqrt{2}\,\sqrt{-3-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cos[x]^2)^(3/2),x]
```

```
[Out] (48*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] - 8*Sqrt[3 + Cos[2*x]]*EllipticF[x
, 1/2] + 6*Sin[2*x] + Sin[4*x])/(12*Sqrt[2]*Sqrt[-3 - Cos[2*x]])
```

Maple [A]
time = 0.47, size = 110, normalized size = 1.24

method	result
default	$\frac{\sqrt{-\left(1+\cos^{2}\left(x\right)\right)\left(\sin^{2}\left(x\right)\right)}\,\left(-\cos(x)(\sin^{4}(x))+10i\sqrt{-\left(\sin^{2}\left(x\right)\right)+2}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\mathrm{EllipticF}(i\cos(x),i)-6\right.}{3\sqrt{\cos^{4}\left(x\right)-1}\,\sin(x)\sqrt{-1-\left(\cos^{2}\left(x\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(-cos(x)*sin(x)^4+10*I*(-sin(x)^2+2)^(1/
2)*(sin(x)^2)^(1/2)*EllipticF(I*cos(x),I)-6*I*(-sin(x)^2+2)^(1/2)*(sin(x)^2
)^(1/2)*EllipticE(I*cos(x),I)+2*sin(x)^2*cos(x))/(cos(x)^4-1)^(1/2)/sin(x)/
(-1-cos(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x)^2 - 1)^(3/2), x)
```

Fricas [F]
time = 0.09, size = 145, normalized size = 1.63

$$\frac{24\left(e^{(4ix)}-e^{(3ix)}\right)\mathrm{integral}\left(-\frac{4\sqrt{e^{(4ix)}+6e^{(2ix)}+1}\,\left(5e^{(2ix)}+2e^{(ix)}+5\right)}{3\left(e^{(6ix)}-2e^{(5ix)}+7e^{(4ix)}-12e^{(3ix)}+7e^{(2ix)}-2e^{(ix)}+1\right)},x\right)-\left(e^{(5ix)}-e^{(4ix)}+24e^{(3ix)}+24e^{(2ix)}-e^{(ix)}+1\right)\sqrt{e^{(4ix)}+6e^{(2ix)}+1}}{24\left(e^{(4ix)}-e^{(3ix)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*(e^(4*I*x) - e^(3*I*x))*integral(-4/3*sqrt(e^(4*I*x) + 6*e^(2*I*x)
 + 1)*(5*e^(2*I*x) + 2*e^(I*x) + 5)/(e^(6*I*x) - 2*e^(5*I*x) + 7*e^(4*I*x)
- 12*e^(3*I*x) + 7*e^(2*I*x) - 2*e^(I*x) + 1), x) - (e^(5*I*x) - e^(4*I*x)
+ 24*e^(3*I*x) + 24*e^(2*I*x) - e^(I*x) + 1)*sqrt(e^(4*I*x) + 6*e^(2*I*x) +
 1))/(e^(4*I*x) - e^(3*I*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\cos^{2}\left(x\right)-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((-cos(x)**2 - 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x)^2 - 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(-\cos\left(x\right)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int((- cos(x)^2 - 1)^(3/2), x)
```

3.60 $\int \left(a+b\cos^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{1+\frac{b\cos^2(x)}{a}}}-\frac{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}\,F\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}}+\frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}$$

```
[Out] 1/3*b*cos(x)*sin(x)*(a+b*cos(x)^2)^(1/2)-2/3*(2*a+b)*(sin(x)^2)^(1/2)/sin(x
)*EllipticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2)/(1+b*cos(x)^2/a)^(1/2
)+1/3*a*(a+b)*(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),(-b/a)^(1/2))*(1+b*c
os(x)^2/a)^(1/2)/(a+b*cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3259, 3251, 3257, 3256, 3262, 3261}

$$\frac{1}{3}b\sin(x)\cos(x)\sqrt{a+b\cos^2(x)}-\frac{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}\,F\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{3\sqrt{a+b\cos^2(x)}}+\frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{3\sqrt{\frac{b\cos^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(3/2),x]
```

```
[Out] (2*(2*a + b)*Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/(3*Sqrt[1 +
(b*Cos[x]^2)/a]) - (a*(a + b)*Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x,
-(b/a)])/(3*Sqrt[a + b*Cos[x]^2]) + (b*Cos[x]*Sqrt[a + b*Cos[x]^2]*Sin[x])/
3
```

Rule 3251

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3259

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p - 1)/(2*f*p)), x] + Dis
t[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a +
 b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a
 + b, 0] && GtQ[p, 1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cos^2(x)\right)^{3/2} dx &= \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\,\sin(x)+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\cos^2(x)}{\sqrt{a+b\cos^2(x)}}\,dx\\ &= \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\,\sin(x)-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx+\frac{1}{3}(2(2a\\ &= \frac{1}{3}b\cos(x)\sqrt{a+b\cos^2(x)}\,\sin(x)+\frac{\left(2(2a+b)\sqrt{a+b\cos^2(x)}\right)\int \sqrt{1+\frac{b\cos^2(x}{a}}}{3\sqrt{1+\frac{b\cos^2(x)}{a}}}\\ &= \frac{2(2a+b)\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{3\sqrt{1+\frac{b\cos^2(x)}{a}}}-\frac{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}\,F\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right.}{3\sqrt{a+b\cos^2(x)}}\end{aligned}$$

Mathematica [A]

time = 0.53, size = 123, normalized size = 1.02

$$\frac{8(2a^2+3ab+b^2)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right)-4a(a+b)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,F\left(x\middle|\frac{b}{a+b}\right)+\sqrt{2}\,b(2a+b+b\cos(2x))\sin(2x)}{12\sqrt{2a+b+b\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(3/2),x]
```

```
[Out] (8*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x,
b/(a + b)] - 4*a*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x,
b/(a + b)] + Sqrt[2]*b*(2*a + b + b*Cos[2*x])*Sin[2*x])/(12*Sqrt[2*a + b +
b*Cos[2*x]])
```

Maple [A]
time = 0.41, size = 192, normalized size = 1.59

method	result
default	$-\frac{-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{\frac{a+b(\cos^2(x))}{a}}\,\text{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{\frac{a+b(\cos^2(x))}{a}}\,\text{EllipticF}\left(\cos(x),\sqrt{-\right.}}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-1/a*b)^
(1/2))*a^2-1/3*a*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x)
,(-1/a*b)^(1/2))*b+4/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticE(
cos(x),(-1/a*b)^(1/2))*a^2+2/3*(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*El
lipticE(cos(x),(-1/a*b)^(1/2))*a*b+1/3*cos(x)^5*b^2+1/3*a*b*cos(x)^3-1/3*b^
2*cos(x)^3-1/3*a*b*cos(x))/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x)^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.13, size = 12, normalized size = 0.10

$$\text{integral}\left(\left(b\cos(x)^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(x)^2 + a)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\cos^2\left(x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cos(x)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(x)^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\cos\left(x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)^(3/2),x)
```

```
[Out] int((a + b*cos(x)^2)^(3/2), x)
```

3.61 $\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$F\left(\frac{\pi}{2}+x\middle|-1\right)$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),I)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{3261\}$

$$F\left(x+\frac{\pi}{2}\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] EllipticF[Pi/2 + x, -1]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx = F\left(\frac{\pi}{2}+x\middle|-1\right)$$

Mathematica [A]
time = 0.04, size = 11, normalized size = 1.22

$$\frac{F\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] EllipticF[x, 1/2]/Sqrt[2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.
time = 0.32, size = 41, normalized size = 4.56

method	result	size
default	$-\frac{\sqrt{\left(1+\cos^{2}\left(x\right)\right)\left(\sin^{2}\left(x\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos\left(2x\right)}{2}}\,\mathrm{EllipticF}\left(\cos\left(x\right),i\right)}{\sqrt{1-\left(\cos^{4}\left(x\right)\right)}\,\sin\left(x\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((1+cos(x)^2)*sin(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(1-cos(x)^4)^(1/2)*Elliptic
F(cos(x),I)/sin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(cos(x)^2 + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(16) = 32$.
time = 0.11, size = 87, normalized size = 9.67

$$\sqrt{2\sqrt{2}-3}\left(2i\sqrt{2}+3i\right)F(\arcsin\left(\sqrt{2\sqrt{2}-3}\left(\cos\left(x\right)+i\sin\left(x\right)\right)\right)\mid 12\sqrt{2}+17)+\sqrt{2\sqrt{2}-3}\left(-2i\sqrt{2}-3i\right)F(\arcsin\left(\sqrt{2\sqrt{2}-3}\left(\cos\left(x\right)-i\sin\left(x\right)\right)\right)\mid 12\sqrt{2}+17)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*sqrt(2) - 3)*(2*I*sqrt(2) + 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) -
3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 17) + sqrt(2*sqrt(2) - 3)*(-2*I*sqrt(
2) - 3*I)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sq
rt(2) + 17)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\cos^{2}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(x)**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(x)^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\sqrt{\cos\left(x\right)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int(1/(cos(x)^2 + 1)^(1/2), x)
```

3.62 $\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1+\cos^2(x)}\,F\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{-1-\cos^2(x)}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),I)*(1+cos(x)^2)^(1/2)/(-1-cos(x)^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3262, 3261}

$$\frac{\sqrt{\cos^2(x)+1}\,F\left(x+\frac{\pi}{2}\middle|-1\right)}{\sqrt{-\cos^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Cos[x]^2],x]
```

```
[Out] (Sqrt[1 + Cos[x]^2]*EllipticF[Pi/2 + x, -1])/Sqrt[-1 - Cos[x]^2]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\cos^2(x)}}\,dx &= \frac{\sqrt{1+\cos^2(x)}\int \frac{1}{\sqrt{1+\cos^2(x)}}\,dx}{\sqrt{-1-\cos^2(x)}}\\ &= \frac{\sqrt{1+\cos^2(x)}\,F\left(\frac{\pi}{2}+x\middle|-1\right)}{\sqrt{-1-\cos^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 1.03

$$\frac{\sqrt{3+\cos(2x)}\,F\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-3-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Cos[x]^2],x]
[Out] (Sqrt[3 + Cos[2*x]]*EllipticF[x, 1/2])/(Sqrt[2]*Sqrt[-3 - Cos[2*x]])
```

Maple [A]
time = 0.28, size = 62, normalized size = 1.94

method	result	size
default	$\frac{i\sqrt{-\left(1+\cos^{2}(x)\right)\left(\sin^{2}(x)\right)}\,\sqrt{1+\cos^{2}(x)}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\mathrm{EllipticF}(i\cos(x),i)}{\sqrt{\cos^{4}(x)-1}\,\sin(x)\sqrt{-1-\left(\cos^{2}(x)\right)}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(-(1+cos(x)^2)*sin(x)^2)^(1/2)*(1+cos(x)^2)^(1/2)*(sin(x)^2)^(1/2)/(cos(x
)^4-1)^(1/2)*EllipticF(I*cos(x),I)/sin(x)/(-1-cos(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-cos(x)^2 - 1), x)
```

Fricas [A]
time = 0.08, size = 41, normalized size = 1.28

$$2\left(2\sqrt{2}+3\right)\sqrt{2\sqrt{2}-3}\,F(\arcsin\left(\sqrt{2\sqrt{2}-3}\,e^{(ix)}\right)\mid 12\sqrt{2}+17)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] 2*(2*sqrt(2) + 3)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)
*e^(I*x)), 12*sqrt(2) + 17)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos^2(x) - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cos(x)**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-cos(x)^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-{\cos(x)}^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cos(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(- cos(x)^2 - 1)^(1/2), x)
```

3.63 $\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{1+\frac{b\cos^2(x)}{a}}\,F\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

```
[Out] -(sin(x)^2)^(1/2)/sin(x)*EllipticF(cos(x),(-b/a)^(1/2))*(1+b*cos(x)^2/a)^(1
/2)/(a+b*cos(x)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3262, 3261}

$$\frac{\sqrt{\frac{b\cos^2(x)}{a}+1}\,F\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[1 + (b*Cos[x]^2)/a]*EllipticF[Pi/2 + x, -(b/a)])/Sqrt[a + b*Cos[x]^2]
```

Rule 3261

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1/(S
qrt[a]*f))*EllipticF[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a,
 0]
```

Rule 3262

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + b*(Sin[e + f*x]^2/a)]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx = \frac{\sqrt{1+\frac{b\cos^2(x)}{a}}\int \frac{1}{\sqrt{1+\frac{b\cos^2(x)}{a}}}\,dx}{\sqrt{a+b\cos^2(x)}}$$
$$= \frac{\sqrt{1+\frac{b\cos^2(x)}{a}}\,F\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{\sqrt{a+b\cos^2(x)}}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 1.10

$$\frac{\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,F\left(x\middle|\frac{b}{a+b}\right)}{\sqrt{2a+b+b\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] (Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticF[x, b/(a + b)])/Sqrt[2*a + b
 + b*Cos[2*x]]
```

Maple [A]
time = 0.27, size = 48, normalized size = 1.14

method	result	size
default	$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{\frac{a+b(\cos^2(x))}{a}}\,\text{EllipticF}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)}{\sin(x)\sqrt{a+b\left(\cos^2\left(x\right)\right)}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(sin(x)^2)^(1/2)*((a+b*cos(x)^2)/a)^(1/2)*EllipticF(cos(x),(-1/a*b)^(1/2))
/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(x)^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 276, normalized size = 6.57

$$-\frac{\left(-2 i b \sqrt{\frac{a^2+ab}{b^2}}-2 i a-i b\right) \sqrt{b} \sqrt{\frac{2 b \sqrt{\frac{a^2+ab}{b^2}}-2 a-b}{b}} F(\arcsin\left(\sqrt{\frac{2 b \sqrt{\frac{a^2+ab}{b^2}}-2 a-b}{b}} (\cos(x)+i \sin(x))\right) \mid \frac{8 a^2+8 ab+b^2+4 (2 ab+b^2) \sqrt{\frac{a^2+ab}{b^2}}}{b^2})+\left(2 i b \sqrt{\frac{a^2+ab}{b^2}}+2 i a+i b\right) \sqrt{b} \sqrt{\frac{2 b \sqrt{\frac{a^2+ab}{b^2}}-2 a-b}{b}} F(\arcsin\left(\sqrt{\frac{2 b \sqrt{\frac{a^2+ab}{b^2}}-2 a-b}{b}} (\cos(x)-i \sin(x))\right) \mid \frac{8 a^2+8 ab+b^2+4 (2 ab+b^2) \sqrt{\frac{a^2+ab}{b^2}}}{b^2})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -((-2*I*b*sqrt((a^2 + a*b)/b^2) - 2*I*a - I*b)*sqrt(b)*sqrt((2*b*sqrt((a^2
+ a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2)
 - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)
*sqrt((a^2 + a*b)/b^2))/b^2) + (2*I*b*sqrt((a^2 + a*b)/b^2) + 2*I*a + I*b)*
sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqr
t((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) - I*sin(x))), (8*a^2 + 8
*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\cos^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\cos(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*cos(x)^2)^(1/2), x)
```

3.64 $\int \frac{1}{\left(1+\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}E\left(\frac{\pi}{2}+x\middle|-1\right)-\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}$$

```
[Out] -1/2*(sin(x)^2)^(1/2)/sin(x)*EllipticE(cos(x),I)-1/2*cos(x)*sin(x)/(1+cos(x
)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3263, 21, 3256}

$$\frac{1}{2}E\left(x+\frac{\pi}{2}\middle|-1\right)-\frac{\sin(x)\cos(x)}{2\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] EllipticE[Pi/2 + x, -1]/2 - (Cos[x]*Sin[x])/(2*Sqrt[1 + Cos[x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(1+\cos^2(x))^{3/2}}\,dx = -\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}} - \frac{1}{2}\int \frac{-1-\cos^2(x)}{\sqrt{1+\cos^2(x)}}\,dx$$
$$= -\frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}} + \frac{1}{2}\int \sqrt{1+\cos^2(x)}\,dx$$
$$= \frac{1}{2}E\left(\frac{\pi}{2}+x\middle|-1\right) - \frac{\cos(x)\sin(x)}{2\sqrt{1+\cos^2(x)}}$$

Mathematica [**A**]
time = 0.07, size = 35, normalized size = 1.09

$$\frac{E\left(x\middle|\frac{1}{2}\right)}{\sqrt{2}} - \frac{\sin(2x)}{2\sqrt{2}\sqrt{3+\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-3/2),x]
```

```
[Out] EllipticE[x, 1/2]/Sqrt[2] - Sin[2*x]/(2*Sqrt[2]*Sqrt[3 + Cos[2*x]])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(32) = 64.
time = 0.43, size = 70, normalized size = 2.19

method	result	s
default	$-\frac{\sqrt{-(\sin^4(x))+2(\sin^2(x))}\left(\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\sqrt{-(\sin^2(x))+2}\,\mathrm{EllipticE}(\cos(x),i)+(\sin^2(x))\cos(x)\right)}{2\sqrt{1-(\cos^4(x))}\,\sin(x)\sqrt{1+\cos^2(x)}}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-sin(x)^4+2*sin(x)^2)^(1/2)*((sin(x)^2)^(1/2)*(-sin(x)^2+2)^(1/2)*Ell
ipticE(cos(x),I)+sin(x)^2*cos(x))/(1-cos(x)^4)^(1/2)/sin(x)/(1+cos(x)^2)^(1
/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x)^2 + 1)^(-3/2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(31) = 62$.
time = 0.13, size = 247, normalized size = 7.72

$$\frac{\left(\left(2i\sqrt{2}-3i\right)\cos(x)^2+2i\sqrt{2}-3i\right)\sqrt{2\sqrt{2}-3}\,E(\arcsin\left(\sqrt{2\sqrt{2}-3}\,(\cos(x)+i\sin(x))\right)\,|\,12\sqrt{2}+17)+\left(\left(-2i\sqrt{2}+3i\right)\cos(x)^2-2i\sqrt{2}+3i\right)\sqrt{2\sqrt{2}-3}\,E(\arcsin\left(\sqrt{2\sqrt{2}-3}\,(\cos(x)-i\sin(x))\right)\,|\,12\sqrt{2}+17)-4\left(\left(-i\sqrt{2}-3i\right)\cos(x)^2-i\sqrt{2}-3i\right)\sqrt{2\sqrt{2}-3}\,F(\arcsin\left(\sqrt{2\sqrt{2}-3}\,(\cos(x)+i\sin(x))\right)\,|\,12\sqrt{2}+17)-4\left(\left(i\sqrt{2}+3i\right)\cos(x)^2+i\sqrt{2}+3i\right)\sqrt{2\sqrt{2}-3}\,F(\arcsin\left(\sqrt{2\sqrt{2}-3}\,(\cos(x)-i\sin(x))\right)\,|\,12\sqrt{2}+17)-2\sqrt{\cos(x)^2+1}\,\cos(x)\sin(x)}{4\left(\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(((2*I*sqrt(2) - 3*I)*cos(x)^2 + 2*I*sqrt(2) - 3*I)*sqrt(2*sqrt(2) - 3)
*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 1
7) + ((-2*I*sqrt(2) + 3*I)*cos(x)^2 - 2*I*sqrt(2) + 3*I)*sqrt(2*sqrt(2) - 3
)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(2) +
17) - 4*((-I*sqrt(2) - 3*I)*cos(x)^2 - I*sqrt(2) - 3*I)*sqrt(2*sqrt(2) - 3)
*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) + I*sin(x))), 12*sqrt(2) + 1
7) - 4*((I*sqrt(2) + 3*I)*cos(x)^2 + I*sqrt(2) + 3*I)*sqrt(2*sqrt(2) - 3)*e
lliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*(cos(x) - I*sin(x))), 12*sqrt(2) + 17)
 - 2*sqrt(cos(x)^2 + 1)*cos(x)*sin(x))/(cos(x)^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\cos^2(x)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2)**(3/2),x)
```

```
[Out] Integral((cos(x)**2 + 1)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x)^2 + 1)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(\cos(x)^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1)^(3/2),x)
[Out] int(1/(cos(x)^2 + 1)^(3/2), x)
```

3.65 $\int \frac{1}{\left(-1-\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{2\sqrt{1+\cos^2(x)}}+\frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}$$

```
[Out] 1/2*cos(x)*sin(x)/(-1-cos(x)^2)^(1/2)-1/2*(sin(x)^2)^(1/2)/sin(x)*EllipticE
(cos(x),I)*(-1-cos(x)^2)^(1/2)/(1+cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3263, 21, 3257, 3256}

$$\frac{\sin(x)\cos(x)}{2\sqrt{-\cos^2(x)-1}}+\frac{\sqrt{-\cos^2(x)-1}\,E\left(x+\frac{\pi}{2}\middle|-1\right)}{2\sqrt{\cos^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Cos[x]^2)^(-3/2),x]
```

```
[Out] (Sqrt[-1 - Cos[x]^2]*EllipticE[Pi/2 + x, -1])/(2*Sqrt[1 + Cos[x]^2]) + (Cos
[x]*Sin[x])/(2*Sqrt[-1 - Cos[x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
```

```
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1-\cos^2(x)\right)^{3/2}}\,dx &= \frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}-\frac{1}{2}\int \frac{1+\cos^2(x)}{\sqrt{-1-\cos^2(x)}}\,dx\\
&= \frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}+\frac{1}{2}\int \sqrt{-1-\cos^2(x)}\,dx\\
&= \frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}+\frac{\sqrt{-1-\cos^2(x)}\int \sqrt{1+\cos^2(x)}\,dx}{2\sqrt{1+\cos^2(x)}}\\
&= \frac{\sqrt{-1-\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-1\right)}{2\sqrt{1+\cos^2(x)}}+\frac{\cos(x)\sin(x)}{2\sqrt{-1-\cos^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.77

$$\frac{-2\sqrt{3+\cos(2x)}\,E\left(x\middle|\frac{1}{2}\right)+\sin(2x)}{2\sqrt{2}\,\sqrt{-3-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Cos[x]^2)^(-3/2),x]
```

```
[Out] (-2*Sqrt[3 + Cos[2*x]]*EllipticE[x, 1/2] + Sin[2*x])/(2*Sqrt[2]*Sqrt[-3 - C
os[2*x]])
```

Maple [A]
time = 0.53, size = 101, normalized size = 1.80

method	result
default	$-\frac{\sqrt{\sin^4(x)-2\left(\sin^2(x)\right)}\left(2i\sqrt{-\left(\sin^2(x)\right)+2}\,\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\mathrm{EllipticF}(i\cos(x),i)-i\sqrt{-\left(\sin^2(x)\right)+2}\right.}{2\sqrt{\cos^4(x)-1}\,\sin(x)\sqrt{-1-\left(\cos^2(x)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(sin(x)^4-2*sin(x)^2)^(1/2)*(2*I*EllipticF(I*cos(x),I)*(sin(x)^2)^(1/2
)*(-sin(x)^2+2)^(1/2)-I*EllipticE(I*cos(x),I)*(sin(x)^2)^(1/2)*(-sin(x)^2+2
)^(1/2)-sin(x)^2*cos(x))/(cos(x)^4-1)^(1/2)/sin(x)/(-1-cos(x)^2)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x)^2 - 1)^(-3/2), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(51) = 102.
time = 0.11, size = 168, normalized size = 3.00

$$-\frac{\left(\left(2\sqrt{2}-3\right)e^{(4ix)}+6\left(2\sqrt{2}-3\right)e^{(2ix)}+2\sqrt{2}-3\right)\sqrt{2\sqrt{2}-3}\,E(\arcsin\left(\sqrt{2\sqrt{2}-3}\,e^{(ix)}\right)\,|\,12\sqrt{2}+17)+4\left(\left(\sqrt{2}+3\right)e^{(4ix)}+6\left(\sqrt{2}+3\right)e^{(2ix)}+\sqrt{2}+3\right)\sqrt{2\sqrt{2}-3}\,F(\arcsin\left(\sqrt{2\sqrt{2}-3}\,e^{(ix)}\right)\,|\,12\sqrt{2}+17)+\sqrt{e^{(4ix)}+6e^{(2ix)}+1}\left(e^{(3ix)}+3e^{(ix)}\right)}{2\left(e^{(4ix)}+6e^{(2ix)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(((2*sqrt(2) - 3)*e^(4*I*x) + 6*(2*sqrt(2) - 3)*e^(2*I*x) + 2*sqrt(2)
- 3)*sqrt(2*sqrt(2) - 3)*elliptic_e(arcsin(sqrt(2*sqrt(2) - 3)*e^(I*x)), 12
*sqrt(2) + 17) + 4*((sqrt(2) + 3)*e^(4*I*x) + 6*(sqrt(2) + 3)*e^(2*I*x) + s
qrt(2) + 3)*sqrt(2*sqrt(2) - 3)*elliptic_f(arcsin(sqrt(2*sqrt(2) - 3)*e^(I*
x)), 12*sqrt(2) + 17) + sqrt(e^(4*I*x) + 6*e^(2*I*x) + 1)*(e^(3*I*x) + 3*e^
(I*x)))/(e^(4*I*x) + 6*e^(2*I*x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\cos^{2}\left(x\right)-1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)**2)**(3/2),x)
```

```
[Out] Integral((-cos(x)**2 - 1)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x)^2 - 1)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-\cos \left(x\right)^2-1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- cos(x)^2 - 1)^(3/2),x)
```

```
[Out] int(1/(- cos(x)^2 - 1)^(3/2), x)
```

3.66 $\int \frac{1}{(a+b\cos^2(x))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}}-\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

```
[Out] -b*cos(x)*sin(x)/a/(a+b)/(a+b*cos(x)^2)^(1/2)-(sin(x)^2)^(1/2)/sin(x)*Ellip
ticE(cos(x),(-b/a)^(1/2))*(a+b*cos(x)^2)^(1/2)/a/(a+b)/(1+b*cos(x)^2/a)^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3263, 21, 3257, 3256}

$$\frac{\sqrt{a+b\cos^2(x)}\,E\left(x+\frac{\pi}{2}\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{\frac{b\cos^2(x)}{a}+1}}-\frac{b\sin(x)\cos(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)^(-3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[x]^2]*EllipticE[Pi/2 + x, -(b/a)])/(a*(a + b)*Sqrt[1 + (b*C
os[x]^2)/a]) - (b*Cos[x]*Sin[x])/(a*(a + b)*Sqrt[a + b*Cos[x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3256

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]/f)*EllipticE[e + f*x, -b/a], x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3257

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + b*(Sin[e + f*x]^2/a)], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3263

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-b)*C
os[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x]^2)^(p + 1)/(2*a*f*(p + 1)*(a
+ b))), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p +
1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\cos^2(x)\right)^{3/2}}\,dx &= -\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}-\frac{\int \frac{-a-b\cos^2(x)}{\sqrt{a+b\cos^2(x)}}\,dx}{a(a+b)}\\ &= -\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}+\frac{\int \sqrt{a+b\cos^2(x)}\,dx}{a(a+b)}\\ &= -\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}+\frac{\sqrt{a+b\cos^2(x)}\int \sqrt{1+\frac{b\cos^2(x)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}}\\ &= \frac{\sqrt{a+b\cos^2(x)}\,E\left(\frac{\pi}{2}+x\middle|-\frac{b}{a}\right)}{a(a+b)\sqrt{1+\frac{b\cos^2(x)}{a}}}-\frac{b\cos(x)\sin(x)}{a(a+b)\sqrt{a+b\cos^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.96

$$\frac{2(a+b)\sqrt{\frac{2a+b+b\cos(2x)}{a+b}}\,E\left(x\middle|\frac{b}{a+b}\right)-\sqrt{2}\,b\sin(2x)}{2a(a+b)\sqrt{2a+b+b\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)^(-3/2),x]
```

```
[Out] (2*(a + b)*Sqrt[(2*a + b + b*Cos[2*x])/(a + b)]*EllipticE[x, b/(a + b)] - S
qrt[2]*b*Sin[2*x])/(2*a*(a + b)*Sqrt[2*a + b + b*Cos[2*x]])
```

Maple [A]
time = 0.40, size = 73, normalized size = 0.94

method	result	size

default	$-\frac{\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}\,\sqrt{-\frac{b(\sin^2(x))}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\cos(x),\sqrt{-\frac{b}{a}}\right)+b\cos(x)(\sin^2(x))}{a(a+b)\sin(x)\sqrt{a+b\left(\cos^2(x)\right)}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((sin(x)^2)^(1/2)*(-b/a*sin(x)^2+(a+b)/a)^(1/2)*a*EllipticE(cos(x),(-1/a*b
)^(1/2))+b*cos(x)*sin(x)^2)/a/(a+b)/sin(x)/(a+b*cos(x)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x)^2 + a)^(-3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.16, size = 775, normalized size = 9.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(b*cos(x)^2 + a)*b^3*cos(x)*sin(x) + (2*I*a^2*b + I*a*b^2 + (2*
I*a*b^2 + I*b^3)*cos(x)^2 - 2*(I*b^3*cos(x)^2 + I*a*b^2)*sqrt((a^2 + a*b)/b
^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_e(arcsi
n(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^
2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + (-2*I*a^2*b
 - I*a*b^2 + (-2*I*a*b^2 - I*b^3)*cos(x)^2 - 2*(-I*b^3*cos(x)^2 - I*a*b^2)*
sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b
)*elliptic_e(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) -
 I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/
b^2) + 2*(-2*I*a^3 - 3*I*a^2*b - I*a*b^2 + (-2*I*a^2*b - 3*I*a*b^2 - I*b^3)
*cos(x)^2 + 2*(-I*a*b^2*cos(x)^2 - I*a^2*b)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*
sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*s
qrt((a^2 + a*b)/b^2) - 2*a - b)/b)*(cos(x) + I*sin(x))), (8*a^2 + 8*a*b + b
^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a*b)/b^2))/b^2) + 2*(2*I*a^3 + 3*I*a^2*b +
 I*a*b^2 + (2*I*a^2*b + 3*I*a*b^2 + I*b^3)*cos(x)^2 + 2*(I*a*b^2*cos(x)^2 +
 I*a^2*b)*sqrt((a^2 + a*b)/b^2))*sqrt(b)*sqrt((2*b*sqrt((a^2 + a*b)/b^2) -
```

```
2*a - b)/b)*elliptic_f(arcsin(sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b)
*(cos(x) - I*sin(x))), (8*a^2 + 8*a*b + b^2 + 4*(2*a*b + b^2)*sqrt((a^2 + a
*b)/b^2))/b^2))/(a^3*b^2 + a^2*b^3 + (a^2*b^3 + a*b^4)*cos(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cos^2\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2)**(3/2),x)
```

```
[Out] Integral((a + b*cos(x)**2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(x)^2 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\cos\left(x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*cos(x)^2)^(3/2), x)
```

3.67 $\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$\text{ArcSin}\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

```
[Out] arcsin(1/2*sin(x)*2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3265, 222}

$$\text{ArcSin}\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/Sqrt[2]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx, x, \sin(x)\right)\\ &= \sin^{-1}\left(\frac{\sin(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\text{ArcSin}\left(\frac{\sin(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/Sqrt[2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.
time = 0.27, size = 33, normalized size = 3.67

method	result	size
default	$-\frac{\sqrt{(1+\cos^2(x))(\sin^2(x))}\,\arcsin(\cos^2(x))}{2\sin(x)\sqrt{1+\cos^2(x)}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((1+cos(x)^2)*sin(x)^2)^(1/2)*arcsin(cos(x)^2)/sin(x)/(1+cos(x)^2)^(1/
2)
```

Maxima [A]
time = 0.47, size = 8, normalized size = 0.89

$$\arcsin\left(\frac{1}{2}\sqrt{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*sqrt(2)*sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(8) = 16$.
time = 0.38, size = 49, normalized size = 5.44

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\cos(x)^2+1}\cos(x)^2\sin(x)-\cos(x)\sin(x)}{\cos(x)^4+\cos(x)^2-1}\right)+\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt(cos(x)^2 + 1)*cos(x)^2*sin(x) - cos(x)*sin(x))/(cos(x)^4 +
 cos(x)^2 - 1)) + 1/2*arctan(sin(x)/cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(cos(x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.
time = 0.43, size = 23, normalized size = 2.56

$$\frac{1}{2}\sqrt{-\sin(x)^2+2}\,\sin(x)+\arcsin\left(\frac{1}{2}\sqrt{2}\,\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-sin(x)^2 + 2)*sin(x) + arcsin(1/2*sqrt(2)*sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos(x)}{\sqrt{\cos(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cos(x)^2 + 1)^(1/2),x)
```

```
[Out] int(cos(x)/(cos(x)^2 + 1)^(1/2), x)
```

3.68 $\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\sin(5+3x)\right)$$

[Out] 1/3*arcsin(1/2*sin(5+3*x))

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3265, 222}

$$\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\sin(3x+5)\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[5 + 3*x]/Sqrt[3 + Cos[5 + 3*x]^2],x]

[Out] ArcSin[Sin[5 + 3*x]/2]/3

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(5+3x)}{\sqrt{3+\cos^2(5+3x)}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(5+3x)\right)\\ &= \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(5+3x)\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 15, normalized size = 1.00

$$\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\sin(5+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5 + 3*x]/Sqrt[3 + Cos[5 + 3*x]^2],x]
```

```
[Out] ArcSin[Sin[5 + 3*x]/2]/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.
time = 0.32, size = 57, normalized size = 3.80

method	result	size
default	$\frac{\sqrt{(3+\cos^2(5+3x))(\sin^2(5+3x))}\ \arcsin\left(-1+\frac{\left(\sin^2(5+3x)\right)}{2}\right)}{6\sin(5+3x)\sqrt{3+\cos^2(5+3x)}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((3+cos(5+3*x)^2)*sin(5+3*x)^2)^(1/2)*arcsin(-1+1/2*sin(5+3*x)^2)/sin(5
+3*x)/(3+cos(5+3*x)^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 11, normalized size = 0.73

$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin\left(3x+5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x + 5))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(11) = 22$.
time = 0.39, size = 89, normalized size = 5.93

$$\frac{1}{6}\arctan\left(\frac{\sqrt{\cos\left(3x+5\right)^2+3}\,\left(\cos\left(3x+5\right)^2+1\right)\sin\left(3x+5\right)-4\cos\left(3x+5\right)\sin\left(3x+5\right)}{\cos\left(3x+5\right)^4+6\cos\left(3x+5\right)^2-3}\right)+\frac{1}{6}\arctan\left(\frac{\sin\left(3x+5\right)}{\cos\left(3x+5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan((sqrt(cos(3*x + 5)^2 + 3)*(cos(3*x + 5)^2 + 1)*sin(3*x + 5) - 4*
cos(3*x + 5)*sin(3*x + 5))/(cos(3*x + 5)^4 + 6*cos(3*x + 5)^2 - 3)) + 1/6*a
rctan(sin(3*x + 5)/cos(3*x + 5))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x+5)}{\sqrt{\cos^2(3x+5)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)**2)**(1/2),x)
```

```
[Out] Integral(cos(3*x + 5)/sqrt(cos(3*x + 5)**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(3*x + 5)/sqrt(cos(3*x + 5)^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cos(3\,x+5)}{\sqrt{{\cos(3\,x+5)}^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x + 5)/(cos(3*x + 5)^2 + 3)^(1/2),x)
```

```
[Out] int(cos(3*x + 5)/(cos(3*x + 5)^2 + 3)^(1/2), x)
```

3.69 $\int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$\sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

```
[Out] arcsinh(1/3*sin(x)*3^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3265, 221}

$$\sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[4 - Cos[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]/Sqrt[3]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{4-\cos^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sin(x)\right) \\ &= \sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\sinh^{-1}\left(\frac{\sin(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[4 - Cos[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]/Sqrt[3]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(8) = 16$.
time = 0.38, size = 53, normalized size = 5.89

method	result	size
default	$-\frac{\sqrt{-\left(\cos^{2}\left(x\right)-4\right)\left(\sin^{2}\left(x\right)\right)}\,\ln\left(-\left(\sin^{2}\left(x\right)\right)+\sqrt{\sin^{4}\left(x\right)+3\left(\sin^{2}\left(x\right)\right)}-\frac{3}{2}\right)}{2\sin\left(x\right)\sqrt{4-\left(\cos^{2}\left(x\right)\right)}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(cos(x)^2-4)*sin(x)^2)^(1/2)*ln(-sin(x)^2+(sin(x)^4+3*sin(x)^2)^(1/2
)-3/2)/sin(x)/(4-cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 8, normalized size = 0.89

$$\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(3)*sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(8) = 16$.
time = 0.39, size = 39, normalized size = 4.33

$$\frac{1}{4}\log\left(8\cos\left(x\right)^{4}-4\left(2\cos\left(x\right)^{2}-5\right)\sqrt{-\cos\left(x\right)^{2}+4}\sin\left(x\right)-40\cos\left(x\right)^{2}+41\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 5)*sqrt(-cos(x)^2 + 4)*sin(x) - 40*cos
(x)^2 + 41)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(x\right)}{\sqrt{-\left(\cos\left(x\right)-2\right)\left(\cos\left(x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(-(cos(x) - 2)*(cos(x) + 2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.
time = 0.44, size = 29, normalized size = 3.22

$$\frac{1}{2}\sqrt{\sin\left(x\right)^{2}+3}\,\sin\left(x\right)-\frac{3}{2}\,\log\left(\sqrt{\sin\left(x\right)^{2}+3}-\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(sin(x)^2 + 3)*sin(x) - 3/2*log(sqrt(sin(x)^2 + 3) - sin(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos\left(x\right)}{\sqrt{4-\cos\left(x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4 - cos(x)^2)^(1/2),x)
```

```
[Out] int(cos(x)/(4 - cos(x)^2)^(1/2), x)
```

3.70 $\int \frac{1}{a+b\cos^4(x)}\,dx$

Optimal. Leaf size=487

$$\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}-\sqrt{2}(a+b)^{3/4}\cot(x)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}-\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\text{ArcTan}\left(\sqrt[4]{\ldots}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{a\ldots}}$$

```
[Out] -1/8*ln((a+b)^(3/4)*cot(x)^2+(a+b)^(1/4)*a^(1/2)-a^(1/4)*cot(x)*2^(1/2)*(a+
b-a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(
1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+1/8*ln((a+b)^(3/4)*cot(x)^2+(a+b)^(1/4
)*a^(1/2)+a^(1/4)*cot(x)*2^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)-
(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)+1/
4*arctan((-(a+b)^(3/4)*cot(x)*2^(1/2)+a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/
2))/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/
(a+b)^(1/4)*2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)-1/4*arctan(((a+b)^(3/4)
*cot(x)*2^(1/2)+a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2))/a^(1/4)/(a+b+a^(1/
2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a
+b+a^(1/2)*(a+b)^(1/2))^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$,
Rules used = {3288, 1183, 648, 632, 210, 642}

$$\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\text{ArcTan}\left(\frac{\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}-\sqrt{2}(a+b)^{3/4}\cot(x)}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\text{ArcTan}\left(\frac{\sqrt{2}(a+b)^{3/4}\cot(x)+\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}-\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left((a+b)^{3/4}\cot^2(x)-\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cot(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a}-\sqrt{a+b}\right)\log\left((a+b)^{3/4}\cot^2(x)+\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}\cot(x)+\sqrt{a}\sqrt[4]{a+b}\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^4)^(-1),x]
```

```
[Out] ((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
- Sqrt[2]*(a + b)^(3/4)*Cot[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])
])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((
Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]] +
Sqrt[2]*(a + b)^(3/4)*Cot[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])])
/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) - ((Sq
rt[a] - Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a + b
 - Sqrt[a]*Sqrt[a + b]]*Cot[x] + (a + b)^(3/4)*Cot[x]^2])/(4*Sqrt[2]*a^(3/4
)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) + ((Sqrt[a] - Sqrt[a + b
])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a
+ b]]*Cot[x] + (a + b)^(3/4)*Cot[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sq
rt[a + b - Sqrt[a]*Sqrt[a + b]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a+b\cos^4(x)}\,dx = -\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a+b)x^4}\,dx, x, \cot(x)\right)$$

$$= -\frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)x}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \cot(x)\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} -$$

$$= -\frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \cot(a\right.}{4(a+b)}$$

$$= \frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\cot(x)+\right.}{4\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}$$

$$= \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\cot(x)\right)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} - \frac{\left(\sqrt{a}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 121, normalized size = 0.25

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+i\sqrt{a}\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+i\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^4)^(-1),x]
```

[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + I*Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + I*Sqrt[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + I*Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[-a + I*Sqrt[a]*Sqrt[b]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. $2(343) = 686$.
time = 0.21, size = 1233, normalized size = 2.53

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^4+256a^3b)\_Z^4+32a^2\_Z^2\right)} \_R \ln\left(\mathrm{e}^{2ix}+\left(\frac{128ia^4}{b}+128ia^3\right)\_R^3+\left(-\frac{32a^3}{b}-32a^2\right)\_R\right.$ $\frac{\left(-a^{\frac{5}{2}}\sqrt{2\sqrt{a^2+ab}-2a}-a^{\frac{3}{2}}\sqrt{2\sqrt{a^2+ab}-2a}\sqrt{a^2+ab}-a^{\frac{3}{2}}\sqrt{2\sqrt{a^2+ab}-2a}\,b-\sqrt{a}\sqrt{2\sqrt{a}\right.}{}$
default	$\frac{}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(x)^4),x,method=_RETURNVERBOSE)

[Out] 1/4/a/b/(a+b)^(3/2)*(1/2*(-a^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*b-a^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a*b)/a^(1/2)*ln(a^(1/2)*tan(x)^2-tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))+2*(2*a^2*b+2*b^2*a+1/2*(-a^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*b-a^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2+(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a*b)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)/a^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan(x)-(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)))+1/4/a/b/(a+b)^(3/2)*(1/2*(a^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*b+a^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b-(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a-(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b-(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2-(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a*b)/a^(1/2)*ln(a^(1/2)*tan(x)^2+tan(x)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)+(a+b)^(1/2))+2*(2*a^2*b+2*b^2*a-1/2*(a^(5/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+a^(3/2)*(2*(a^2+a*b)^(1/2

```
)-2*a)^(1/2)*(a^2+a*b)^(1/2)+a^(3/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*b+a^(1/2
)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*b-(a+b)^(1/2)*(2*(a^2+a*b)^
(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*a-(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2
)*(a^2+a*b)^(1/2)*b-(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a^2-(a+b)^(1/
2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*a*b)*(2*((a+b)*a)^(1/2)-2*a)^(1/2)/a^(1/2)
)/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*tan
(x)+(2*((a+b)*a)^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*((a+b)*a)^(1/2)
+2*a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. 2(344) = 688.
time = 0.53, size = 809, normalized size = 1.66

$$-\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\left(b\cos(x)^2+2\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}-(a^3+a^2b-2(a^3+a^2b)\cos(x)^2)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\right)+\frac{1}{8}\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}\log\left(b\cos(x)^2-2\left(ab\cos(x)\sin(x)+(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{-\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}+1}{a^2+ab}}-(a^3+a^2b-2(a^3+a^2b)\cos(x)^2)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\right)+\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\left(-b\cos(x)^2+2\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}-(a^3+a^2b-2(a^3+a^2b)\cos(x)^2)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\right)-\frac{1}{8}\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}\log\left(-b\cos(x)^2-2\left(ab\cos(x)\sin(x)-(a^4+a^3b)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\cos(x)\sin(x)\right)\sqrt{\frac{(a^2+ab)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}-1}{a^2+ab}}-(a^3+a^2b-2(a^3+a^2b)\cos(x)^2)\sqrt{-\frac{b}{a^5+2a^4b+a^3b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)
)*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^
4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b +
a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt
(-b/(a^5 + 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) +
(a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2
 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - (a^3 + a^2*b
 - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))) + 1/8*sqrt
(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b))*log(-b*c
os(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3
*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))
- 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 +
 2*a^4*b + a^3*b^2))) - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*
b^2)) - 1)/(a^2 + a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) - (a^4 + a^3
*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqr
t(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - (a^3 + a^2*b - 2*(a^3 +
 a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 307, normalized size = 0.63

$$\frac{\left(3\sqrt{a^2+\sqrt{-ab}\,a}\,a^2+4\sqrt{a^2+\sqrt{-ab}\,a}\,ab-3\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,a-4\sqrt{a^2+\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)a+16a^2}}{a}}}\right)\right)|a|}{2(3a^5+7a^4b+4a^3b^2)}+\frac{\left(3\sqrt{a^2-\sqrt{-ab}\,a}\,a^2+4\sqrt{a^2-\sqrt{-ab}\,a}\,ab-3\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,a-4\sqrt{a^2-\sqrt{-ab}\,a}\,\sqrt{-ab}\,b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)a+16a^2}}{a}}}\right)\right)|a|}{2(3a^5+7a^4b+4a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b - 3*sq
rt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)
*b)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)*a
+ 16*a^2))/a)))*abs(a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sq
rt(-a*b)*a)*a^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b - 3*sqrt(a^2 - sqrt(-a*b)*
a)*sqrt(-a*b)*a - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*(pi*floor(x/pi +
 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/a)))*abs(
a)/(3*a^5 + 7*a^4*b + 4*a^3*b^2)
```

Mupad [**B**]
time = 2.66, size = 926, normalized size = 1.90

$$-2\,\mathrm{atanh}\left(\frac{8a^6b\tan(x)\sqrt{-\frac{a^2}{16(a^4+ba^3)}-\frac{\sqrt{-a^3b}}{16(a^4+ba^3)}}}{\frac{2a^9b}{a^4+ba^3}-2a^4b^2-2a^5b+\frac{2a^8b^2}{a^4+ba^3}+\frac{2a^7b\sqrt{-a^3b}}{a^4+ba^3}+\frac{2a^6b^2\sqrt{-a^3b}}{a^4+ba^3}}-\frac{8a^2b\tan(x)\sqrt{-\frac{a^2}{16(a^4+ba^3)}-\frac{\sqrt{-a^3b}}{16(a^4+ba^3)}}}{\frac{2a^5b}{a^4+ba^3}-2ab+\frac{2a^3b\sqrt{-a^3b}}{a^4+ba^3}}+\frac{8a^4b\tan(x)\sqrt{-a^3b}\sqrt{-\frac{a^2}{16(a^4+ba^3)}-\frac{\sqrt{-a^3b}}{16(a^4+ba^3)}}}{\frac{2a^9b}{a^4+ba^3}-2a^4b^2-2a^5b+\frac{2a^8b^2}{a^4+ba^3}+\frac{2a^7b\sqrt{-a^3b}}{a^4+ba^3}+\frac{2a^6b^2\sqrt{-a^3b}}{a^4+ba^3}}\right)\sqrt{-\frac{a^2+\sqrt{-a^3b}}{16(a^4+ba^3)}}-2\,\mathrm{atanh}\left(\frac{8a^2b\tan(x)\sqrt{\frac{\sqrt{-a^3b}}{16(a^4+ba^3)}-\frac{a^2}{16(a^4+ba^3)}}}{2ab-\frac{2a^5b}{a^4+ba^3}+\frac{2a^3b\sqrt{-a^3b}}{a^4+ba^3}}-\frac{8a^6b\tan(x)\sqrt{\frac{\sqrt{-a^3b}}{16(a^4+ba^3)}-\frac{a^2}{16(a^4+ba^3)}}}{2a^5b+2a^4b^2-\frac{2a^9b}{a^4+ba^3}-\frac{2a^8b^2}{a^4+ba^3}+\frac{2a^7b\sqrt{-a^3b}}{a^4+ba^3}+\frac{2a^6b^2\sqrt{-a^3b}}{a^4+ba^3}}+\frac{8a^4b\tan(x)\sqrt{-a^3b}\sqrt{\frac{\sqrt{-a^3b}}{16(a^4+ba^3)}-\frac{a^2}{16(a^4+ba^3)}}}{2a^5b+2a^4b^2-\frac{2a^9b}{a^4+ba^3}-\frac{2a^8b^2}{a^4+ba^3}+\frac{2a^7b\sqrt{-a^3b}}{a^4+ba^3}+\frac{2a^6b^2\sqrt{-a^3b}}{a^4+ba^3}}\right)\sqrt{-\frac{a^2-\sqrt{-a^3b}}{16(a^4+ba^3)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^4),x)
```

```
[Out] - 2*atanh((8*a^6*b*tan(x)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(a
^3*b + a^4)))^(1/2))/((2*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*a^
8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*
(-a^3*b)^(1/2))/(a^3*b + a^4)) - (8*a^2*b*tan(x)*(- a^2/(16*(a^3*b + a^4))
- (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/2))/((2*a^5*b)/(a^3*b + a^4) - 2*a*
b + (2*a^3*b*(-a^3*b)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b)^(1/2
)*(- a^2/(16*(a^3*b + a^4)) - (-a^3*b)^(1/2)/(16*(a^3*b + a^4)))^(1/2))/((2
*a^9*b)/(a^3*b + a^4) - 2*a^4*b^2 - 2*a^5*b + (2*a^8*b^2)/(a^3*b + a^4) + (
2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))/(a^3*b +
 a^4)))*(-(a^2 + (-a^3*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2) - 2*atanh((8*a^2
```

```
*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b + a^4)))^(1/2
))/(2*a*b - (2*a^5*b)/(a^3*b + a^4) + (2*a^3*b*(-a^3*b)^(1/2))/(a^3*b + a^4
)) - (8*a^6*b*tan(x)*((-a^3*b)^(1/2)/(16*(a^3*b + a^4)) - a^2/(16*(a^3*b +
a^4)))^(1/2))/(2*a^5*b + 2*a^4*b^2 - (2*a^9*b)/(a^3*b + a^4) - (2*a^8*b^2)/
(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2))/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b
)^(1/2))/(a^3*b + a^4)) + (8*a^4*b*tan(x)*(-a^3*b)^(1/2)*((-a^3*b)^(1/2)/(1
6*(a^3*b + a^4)) - a^2/(16*(a^3*b + a^4)))^(1/2))/(2*a^5*b + 2*a^4*b^2 - (2
*a^9*b)/(a^3*b + a^4) - (2*a^8*b^2)/(a^3*b + a^4) + (2*a^7*b*(-a^3*b)^(1/2)
)/(a^3*b + a^4) + (2*a^6*b^2*(-a^3*b)^(1/2))/(a^3*b + a^4)))*(-(a^2 - (-a^3
*b)^(1/2))/(16*(a^3*b + a^4)))^(1/2)
```

3.71 $\int \frac{1}{a-b\cos^4(x)}\,dx$

Optimal. Leaf size=101

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] -1/2*arctan(cot(x)*(a^(1/2)-b^(1/2))^(1/2)/a^(1/4))/a^(3/4)/(a^(1/2)-b^(1/2
))^(1/2)-1/2*arctan(cot(x)*(a^(1/2)+b^(1/2))^(1/2)/a^(1/4))/a^(3/4)/(a^(1/2
)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3288, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Cot[x])/a^(1/4)]/(a^(3/4)*Sqrt[Sqrt[a]
 - Sqrt[b]]) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Cot[x])/a^(1/4)]/(2*a^(3/4)*
Sqrt[Sqrt[a] + Sqrt[b]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-b\cos^4(x)}\,dx &= -\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \cot(x)\right)\\ &= -\left(\frac{1}{2}\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \cot(x)\right)\right)-\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right.\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cot(x)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 109, normalized size = 1.08

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+\sqrt{a}\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{-a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{-a+\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cos[x]^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a]*Sqrt[a + Sqrt
[a]*Sqrt[b]]) - ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*Sqr
t[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]])
```

Maple [A]
time = 0.14, size = 72, normalized size = 0.71

method	result

default	$a\left(\frac{\arctan\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2a\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{\operatorname{arctanh}\left(\frac{a\tan(x)}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2a\sqrt{\left(\sqrt{ab}-a\right)a}}\right)$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(256a^4-256a^3b)\_Z^4+32a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{128ia^4}{b}+128ia^3\right)\_R^3+\left(\frac{32a^3}{b}-32a^2\right)\_R$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] a*(1/2/a/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*tan(x)/(((a*b)^(1/2)+a)*a)^(1/2
))-1/2/a/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(a*tan(x)/(((a*b)^(1/2)-a)*a)^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^4 - a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(65) = 130.
time = 0.50, size = 817, normalized size = 8.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b))
*log(b*cos(x)^2 + 2*(a*b*cos(x)*sin(x) - (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*
b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3
*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/
(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqrt(-((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b
+ a^3*b^2)) + 1)/(a^2 - a*b))*log(b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) - (a^4
```

```
- a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 - a*b
)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) + 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a
^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))) + 1/8*sqrt(((a^2 -
 a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 - a*b))*log(-b*cos(x)^2 +
 2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2))*cos
(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a^2 -
 a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^2)*sqrt(b/(a^5 - 2*a^4*b + a
^3*b^2))) - 1/8*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)) - 1)/(a
^2 - a*b))*log(-b*cos(x)^2 - 2*(a*b*cos(x)*sin(x) + (a^4 - a^3*b)*sqrt(b/(a
^5 - 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 - a*b)*sqrt(b/(a^5 - 2*a
^4*b + a^3*b^2)) - 1)/(a^2 - a*b)) + (a^3 - a^2*b - 2*(a^3 - a^2*b)*cos(x)^
2)*sqrt(b/(a^5 - 2*a^4*b + a^3*b^2)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**4),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(65) = 130.
time = 0.44, size = 299, normalized size = 2.96

$$\frac{\left(3\sqrt{a^2+\sqrt{ab}\,a}\,a^2-4\sqrt{a^2+\sqrt{ab}\,a}\,ab-3\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,a+4\sqrt{a^2+\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a}}}\right)\right)|a|}{2\left(3a^5-7a^4b+4a^3b^2\right)}+\frac{\left(3\sqrt{a^2-\sqrt{ab}\,a}\,a^2-4\sqrt{a^2-\sqrt{ab}\,a}\,ab+3\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,a-4\sqrt{a^2-\sqrt{ab}\,a}\,\sqrt{ab}\,b\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a-b)a+16a^2}}{a}}}\right)\right)|a|}{2\left(3a^5-7a^4b+4a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2 - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b - 3*sqrt
(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a + 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b)*(p
i*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a - b)*a + 16*a
^2))/a)))*abs(a)/(3*a^5 - 7*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt(a*b
)*a)*a^2 - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b + 3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a
*b)*a - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*(pi*floor(x/pi + 1/2) + arct
an(2*tan(x)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/a)))*abs(a)/(3*a^5 -
7*a^4*b + 4*a^3*b^2)
```

Mupad [**B**]
time = 2.60, size = 938, normalized size = 9.29

$$2\,\mathrm{atanh}\left(\frac{8a^6b\tan(x)\sqrt{\frac{a^2}{16\,(a^3b-a^4)}+\frac{\sqrt{a^3b}}{16\,(a^3b-a^4)}}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}-\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}+\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}-\frac{8a^2b\tan(x)\sqrt{\frac{a^2}{16\,(a^3b-a^4)}+\frac{\sqrt{a^3b}}{16\,(a^3b-a^4)}}}{2ab+\frac{2a^5b}{a^3b-a^4}+\frac{2a^3b\sqrt{a^3b}}{a^3b-a^4}}+\frac{8a^4b\tan(x)\sqrt{\frac{a^2}{16\,(a^3b-a^4)}+\frac{\sqrt{a^3b}}{16\,(a^3b-a^4)}}\,\sqrt{a^3b}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}-\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}+\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}\right)\sqrt{\frac{a^2+\sqrt{a^3b}}{16\,(a^3b-a^4)}}-2\,\mathrm{atanh}\left(\frac{8a^2b\tan(x)\sqrt{\frac{a^2}{16\,(a^3b-a^4)}-\frac{\sqrt{a^3b}}{16\,(a^3b-a^4)}}}{2ab+\frac{2a^5b}{a^3b-a^4}-\frac{2a^3b\sqrt{a^3b}}{a^3b-a^4}}-\frac{8a^6b\tan(x)\sqrt{\frac{a^2}{16\,(a^3b-a^4)}-\frac{\sqrt{a^3b}}{16\,(a^3b-a^4)}}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}+\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}-\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}+\frac{8a^4b\tan(x)\sqrt{\frac{a^2}{16\,(a^3b-a^4)}-\frac{\sqrt{a^3b}}{16\,(a^3b-a^4)}}\,\sqrt{a^3b}}{2a^5b-2a^4b^2-\frac{2a^8b^2}{a^3b-a^4}+\frac{2a^9b}{a^3b-a^4}+\frac{2a^6b^2\sqrt{a^3b}}{a^3b-a^4}-\frac{2a^7b\sqrt{a^3b}}{a^3b-a^4}}\right)\sqrt{\frac{a^2-\sqrt{a^3b}}{16\,(a^3b-a^4)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^4),x)
```

```
[Out] 2*atanh((8*a^6*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^3*b)^(1/2)/(16*(a^3*b
- a^4)))^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b
)/(a^3*b - a^4) - (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^3*b
)^(1/2))/(a^3*b - a^4)) - (8*a^2*b*tan(x)*(a^2/(16*(a^3*b - a^4)) + (a^3*b)
^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b + (2*a^5*b)/(a^3*b - a^4) + (2*a^3
*b*(a^3*b)^(1/2))/(a^3*b - a^4)) + (8*a^4*b*tan(x)*(a^2/(16*(a^3*b - a^4))
+ (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b - 2*a^4*b
^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) - (2*a^6*b^2*(a^3*
b)^(1/2))/(a^3*b - a^4) + (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4)))*((a^2 + (
a^3*b)^(1/2))/(16*(a^3*b - a^4)))^(1/2) - 2*atanh((8*a^2*b*tan(x)*(a^2/(16*
(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a*b + (2*a^5*b
)/(a^3*b - a^4) - (2*a^3*b*(a^3*b)^(1/2))/(a^3*b - a^4)) - (8*a^6*b*tan(x)*
(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)))^(1/2))/(2*a^5*b
 - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) + (2*a^9*b)/(a^3*b - a^4) + (2*a^6
*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) - (2*a^7*b*(a^3*b)^(1/2))/(a^3*b - a^4))
+ (8*a^4*b*tan(x)*(a^2/(16*(a^3*b - a^4)) - (a^3*b)^(1/2)/(16*(a^3*b - a^4)
))^(1/2)*(a^3*b)^(1/2))/(2*a^5*b - 2*a^4*b^2 - (2*a^8*b^2)/(a^3*b - a^4) +
(2*a^9*b)/(a^3*b - a^4) + (2*a^6*b^2*(a^3*b)^(1/2))/(a^3*b - a^4) - (2*a^7*
b*(a^3*b)^(1/2))/(a^3*b - a^4)))*((a^2 - (a^3*b)^(1/2))/(16*(a^3*b - a^4)))
^(1/2)
```

3.72 $\int \frac{1}{1+\cos^4(x)}\,dx$

Optimal. Leaf size=292

$$\frac{x}{2\sqrt{-1+\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{\left(-2+\sqrt{2}\right)\cos(x)\sin(x)+\sqrt{-1+\sqrt{2}}\,(1-2\sin^2(x))}{2+\sqrt{1+\sqrt{2}}+2\sqrt{-1+\sqrt{2}}\,\cos(x)\sin(x)+\left(-2+\sqrt{2}\right)\sin^2(x)}\right)}{4\sqrt{-1+\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{\left(-2+\sqrt{}\right.}{2+\sqrt{1+\sqrt{}}}\right.}{}$$

[Out] 1/2*x/(2^(1/2)-1)^(1/2)+1/4*arctan((cos(x)*sin(x)*(-2+2^(1/2))+(-1+2*sin(x)^2)*(2^(1/2)-1)^(1/2))/(2+sin(x)^2*(-2+2^(1/2))-2*cos(x)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)+1/4*arctan((cos(x)*sin(x)*(-2+2^(1/2))+(1-2*sin(x)^2)*(2^(1/2)-1)^(1/2))/(2+sin(x)^2*(-2+2^(1/2))+2*cos(x)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)+1/8*ln(2*cot(x)^2+2^(1/2)-2*cot(x)*(2^(1/2)-1)^(1/2))*(2^(1/2)-1)^(1/2)-1/8*ln(1+cot(x)^2*2^(1/2)+cot(x)*(-2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)

Rubi [A]
time = 0.13, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3288, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2}-1}\,(1-2\sin^2(x))+\left(\sqrt{2}-2\right)\sin(x)\cos(x)}{\left(\sqrt{2}-2\right)\sin^2(x)+2\sqrt{\sqrt{2}-1}\,\sin(x)\cos(x)+\sqrt{1+\sqrt{2}}+2}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt{2}-1}\,(2\sin^2(x)-1)+\left(\sqrt{2}-2\right)\sin(x)\cos(x)}{\left(\sqrt{2}-2\right)\sin^2(x)-2\sqrt{\sqrt{2}-1}\,\sin(x)\cos(x)+\sqrt{1+\sqrt{2}}+2}\right)}{4\sqrt{\sqrt{2}-1}}+\frac{x}{2\sqrt{\sqrt{2}-1}}+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(2\cot^2(x)-2\sqrt{\sqrt{2}-1}\cot(x)+\sqrt{2}\right)-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}\cot^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\cot(x)+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x]^4)^(-1),x]

[Out] x/(2*Sqrt[-1 + Sqrt[2]]) + ArcTan[((-2 + Sqrt[2])*Cos[x]*Sin[x] + Sqrt[-1 + Sqrt[2]]*(1 - 2*Sin[x]^2))/(2 + Sqrt[1 + Sqrt[2]] + 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x] + (-2 + Sqrt[2])*Sin[x]^2)]/(4*Sqrt[-1 + Sqrt[2]]) + ArcTan[((-2 + Sqrt[2])*Cos[x]*Sin[x] + Sqrt[-1 + Sqrt[2]]*(-1 + 2*Sin[x]^2))/(2 + Sqrt[1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x] + (-2 + Sqrt[2])*Sin[x]^2)]/(4*Sqrt[-1 + Sqrt[2]]) + (Sqrt[-1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Cot[x] + 2*Cot[x]^2])/8 - (Sqrt[-1 + Sqrt[2]]*Log[1 + Sqrt[2*(-1 + Sqrt[2])]*Cot[x] + Sqrt[2]*Cot[x]^2])/8

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{1+\cos^4(x)}\,dx = -\text{Subst}\left(\int \frac{1+x^2}{1+2x^2+2x^4}\,dx, x, \cot(x)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}-\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \cot(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}+\left(1-\frac{1}{\sqrt{2}}\right.}{\frac{1}{\sqrt{2}}+\sqrt{-1+\sqrt{2}}\,x+}\right.}{2\sqrt{2\left(-1+\sqrt{2}\right.}}$$

$$= \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{Subst}\left(\int \frac{-\sqrt{-1+\sqrt{2}}+2x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \cot(x)\right) - \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{S}$$

$$= \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\,\cot(x)+2\cot^2(x)\right) - \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\log\left(1\right.$$

$$= \frac{1}{2}\sqrt{1+\sqrt{2}}\,x - \frac{1}{4}\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\left(2-\sqrt{2}\right)\cos(x)\sin(x)-\sqrt{-1+\sqrt{2}}\,(}{2+\sqrt{1+\sqrt{2}}+2\sqrt{-1+\sqrt{2}}\,\cos(x)\sin(x)-}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 45, normalized size = 0.15

$$\frac{\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{2\sqrt{1-i}} + \frac{\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^4)^(-1),x]
[Out] ArcTan[Tan[x]/Sqrt[1 - I]]/(2*Sqrt[1 - I]) + ArcTan[Tan[x]/Sqrt[1 + I]]/(2*
Sqrt[1 + I])
```

Maple [A]
time = 0.21, size = 167, normalized size = 0.57

method	result
risch	$\frac{\sqrt{-2+2i}\,\ln\left(e^{2ix}-i\sqrt{-2+2i}-\sqrt{-2+2i}+1-2i\right)}{8} - \frac{\sqrt{-2+2i}\,\ln\left(e^{2ix}+i\sqrt{-2+2i}+\sqrt{-2+2i}+1-\right.}{8}$

default	$-\frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{2\left(-1-\sqrt{2}\right)\arctan\left(\frac{2\tan(x)-\sqrt{-2+2\sqrt{2}}}{\sqrt{2\sqrt{2}+2}}\right.}{\sqrt{2\sqrt{2}+2}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+cos(x)^4),x,method=_RETURNVERBOSE)

[Out] -1/8*2^(1/2)*(-1/2*(-2+2*2^(1/2))^(1/2)*ln(tan(x)^2-tan(x)*(-2+2*2^(1/2))^(1/2)+2^(1/2))+2*(-1-2^(1/2))/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)-(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2)))-1/8*2^(1/2)*(1/2*(-2+2*2^(1/2))^(1/2)*ln(tan(x)^2+tan(x)*(-2+2*2^(1/2))^(1/2)+2^(1/2))+2*(-1-2^(1/2))/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)+(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2)))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(x)^4),x, algorithm="maxima")

[Out] integrate(1/(cos(x)^4 + 1), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3830 vs. 2(219) = 438.
time = 18.44, size = 3830, normalized size = 13.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(x)^4),x, algorithm="fricas")

[Out] -1/32*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)^4 + 4*(sqrt(2) - 1)*cos(x)^2 + (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 1) + 1/32*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)^4 + 4*(sqrt(2) - 1)*cos(x)^2 - (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 1) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6

```
*sqrt(2) + 1) - 2*sqrt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2
*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4
)*(4*sqrt(2) + 5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*
sqrt(2) + 58))*cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(
2) - 32))*cos(x)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) -
 2))*cos(x)^7 + 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*co
s(x)^5 - (2*2^(3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^
(3/4)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 + (16*(sqrt(2)*(5*sqr
t(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt
(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)
^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)
*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2
) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8
*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(
3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)
*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*s
qrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2
) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) -
 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*s
qrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)
*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos
(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*
cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*
cos(x)^2 - 1)) + 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*
(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 22)
- 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) - 37
)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^10 -
 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(49*s
qrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) - 2*sq
rt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2)
+ 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) + 5))*c
os(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*cos(x
)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x)^9 -
 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 + 2*(2
^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(3/4)*
(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*sqrt(2
*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2
) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 +
 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7
*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) -
 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*
cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2)
 - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11
) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) -
 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/
```

```
4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(
4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2)
- 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) -
 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 + 4*(2^(1/
4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2)
+ 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(
x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) - 1/1
6*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*
sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 22) - 8*sqrt(2) - 52)*co
s(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) - 37)*cos(x)^12 + 16*(2*
sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 170, normalized size = 0.58

$$\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan\left(x\right)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan\left(x\right)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan\left(x\right)^{2}+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan\left(x\right)+\sqrt{2}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\tan\left(x\right)^{2}-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan\left(x\right)+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2)
+ 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x/pi + 1/
2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sqrt(
2) + 2)))*sqrt(sqrt(2) + 1) - 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^2 + 2^(1/4)*
sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^2 -
 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2))
```

Mupad [**B**]
time = 2.73, size = 214, normalized size = 0.73

$$\mathrm{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}+\frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)-\mathrm{atanh}\left(\frac{4\sqrt{2}\tan(x)\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}-\frac{4\sqrt{2}\tan(x)\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}{64\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+1}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}+2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4 + 1),x)
```

```
[Out] atanh((4*2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64
)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1) + (4*2^(1/2)*tan(x)*(2^(1/2)/64 -
1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) - 1)
)*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2)) - atanh((4*
2^(1/2)*tan(x)*(- 2^(1/2)/64 - 1/64)^(1/2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(
- 2^(1/2)/64 - 1/64)^(1/2) + 1) - (4*2^(1/2)*tan(x)*(2^(1/2)/64 - 1/64)^(1/
2))/(64*(2^(1/2)/64 - 1/64)^(1/2)*(- 2^(1/2)/64 - 1/64)^(1/2) + 1))*(2*(- 2
^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))
```

3.73 $\int \frac{1}{1-\cos^4(x)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{2\sqrt{2}}-\frac{\cot(x)}{2}$$

```
[Out] -1/2*cot(x)+1/4*x*2^(1/2)-1/4*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3288, 396, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{2\sqrt{2}}+\frac{x}{2\sqrt{2}}-\frac{\cot(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[2]) - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/(2*Sqrt[2]
) - Cot[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3288

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-\cos^4(x)}\,dx &= -\text{Subst}\left(\int \frac{1+x^2}{1+2x^2}\,dx, x, \cot(x)\right)\\
&= -\frac{\cot(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(x)\right)\\
&= \frac{x}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{2\sqrt{2}} - \frac{\cot(x)}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 24, normalized size = 0.53

$$\frac{1}{4}\left(\sqrt{2}\,\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right) - 2\cot(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/4
```

Maple [A]
time = 0.06, size = 21, normalized size = 0.47

method	result	size
default	$-\frac{1}{2\tan(x)} + \frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{4}$	21
risch	$-\frac{i}{e^{2ix}-1} + \frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{8} - \frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{8}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/tan(x)+1/4*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.48, size = 20, normalized size = 0.44

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - \frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/2/tan(x)
```

Fricas [A]
time = 0.41, size = 43, normalized size = 0.96

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)\sin(x)+4\cos(x)}{8\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))*si
n(x) + 4*cos(x))/sin(x)
```

Sympy [A]
time = 0.63, size = 78, normalized size = 1.73

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{4}+\frac{\tan\left(\frac{x}{2}\right)}{4}-\frac{1}{4\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**4),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/4 + sqrt(2
)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/4 + tan(x/2)/4 -
 1/(4*tan(x/2))
```

Giac [A]
time = 0.40, size = 53, normalized size = 1.18

$$\frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - 1/2/tan(x)
```

Mupad [B]
time = 2.16, size = 20, normalized size = 0.44

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{4}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^4 - 1),x)
[Out] (2^(1/2)*atan((2^(1/2)*tan(x))/2))/4 - 1/(2*tan(x))
```

3.74 $\int \frac{1}{a+b\cos^5(x)}\,dx$

Optimal. Leaf size=494

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}}}{\sqrt{\sqrt[5]{}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}}}$$

```
[Out] 2/5*arctan((a^(1/5)-b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+b^(1/5))^(1/2))/a^(4
/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1
)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2))/a^(4/
5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2)+2/
5*arctan((a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(2/5)*
b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(2
/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)*tan(1/2*x)
/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(
1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(4/5)*b^(1
/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-
(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3292, 2738, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a^(1/5) - b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + b^(1/5)]])/(5*a
^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt[a
^(1/5) + (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]])
/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5)*b^
(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1
/5) + (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*S
qrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^(3/5)*b
^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1
/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2*ArcTan[(
Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(4/5)*b^(1
/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(4
/5)*b^(1/5)])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\cos^5(x)}\,dx = \int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)\right)}-\frac{}{5a^{4/5}\left(-\sqrt[5]{}\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}-$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}+\left(-\sqrt[5]{a}+\sqrt[5]{b}\right)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{2\text{Subst}\left(\int \frac{}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.22, size = 130, normalized size = 0.26

$$\frac{8}{5}\text{RootSum}\left[b+5b\#1^2+10b\#1^4+32a\#1^5+10b\#1^6+5b\#1^8+b\#1^{10}\&,\frac{2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b+4b\#1^2+16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x]^5)^(-1),x]
```

```
[Out] (8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b*
#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 + #1
^2]*#1^3)/(b + 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.52, size = 150, normalized size = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a-b)\_Z^{10}+(5a+5b)\_Z^{8}+(10a-10b)\_Z^{6}+(10a+10b)\_Z^{4}+(5a-5b)\_Z^{2}+a+b\right)} \frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\right.}{\_R^{9}a-\_R^{9}b+4\_R^{7}a+4\_R^{7}b+6\_R^{5}}\right.}{5}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)} \_R\ln\left(\mathrm{e}^{ix}+\left(-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a-_R^9*b+4*_R^7*a+4*_R^7*b+6*_R
^5*a-6*_R^5*b+4*_R^3*a+4*_R^3*b+_R*a-_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a-b
)*_Z^10+(5*a+5*b)*_Z^8+(10*a-10*b)*_Z^6+(10*a+10*b)*_Z^4+(5*a-5*b)*_Z^2+a+b
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^5 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cos^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**5),x)
```

```
[Out] Integral(1/(a + b*cos(x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*cos(x)^5 + a), x)
```

Mupad [B]
time = 8.82, size = 1520, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^5),x)
```

```
[Out] symsum(log(-(10995116277760*b^7*(a - b)*(7*cot(x/2) - 56*root(9765625*a^8*b
^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d
^4 - 125*a^2*d^2 - 1, d, k)*a + root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^
10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d,
k)*b - 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 - 225000*ro
ot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*
d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 - 3875000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 - 25000000*root(9765625*a^8*b^2*d^10 -
 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*
a^2*d^2 - 1, d, k)^9*a^9 + 735*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1
0 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k
)^2*a^2*cot(x/2) + 28875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 19
53125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^
4*cot(x/2) + 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125
*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^6*cot
```

```
(x/2) + 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^8*cot(x/2
) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b + 100000*root
(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^
6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b + 4000000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b + 50000000*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^9*a^8*b - 125000*root(9765625*a^8*b^2*d^10 - 9765625*a
^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 -
1, d, k)^7*a^5*b^2 - 25000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^9*a^7*b^2 - 35*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a*b*cot(x/2
) - 7000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^3*b*cot(x/2) - 3
50000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156
250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b*cot(x/2) - 5000
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 15625
0*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b*cot(x/2) + 3125*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*cot(x/2) + 1718750*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*cot(x/2)))/cot(x/2))
*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

3.75 $\int \frac{1}{a+b\cos^6(x)}\,dx$

Optimal. Leaf size=171

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] -1/3*arctan(cot(x)*(a^(1/3)+b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/3
))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)/a^(1/6))/a^(5
/6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)+(-1)^(2/3
)*b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3290, 3260, 209}

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^6)^(-1),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Cot[x])/a^(1/6)]/(a^(5/6)*Sqrt[a^(1/3)
 + b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)]/
(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) + (-1
)^(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1
/3)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\cos^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \cot(x)\right)}{3a} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \cot(x\right.}{3a}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\sqrt{\right.}{3a^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.24, size = 146, normalized size = 0.85

$$\frac{8}{3}\operatorname{RootSum}\left[b+6b\#1+15b\#1^2+64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5+b\#1^6\&, \frac{2\operatorname{ArcTan}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{b+5b\#1+32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^6)^(-1),x]
```

```
[Out] (8*RootSum[b + 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*b
*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*C
os[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 + 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 +
5*b*#1^4 + b*#1^5) & ])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.43, size = 60, normalized size = 0.35

method	result

default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+3a\_Z^{2}+a+b\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{5}+2\_R^{3}+\_R}}{6a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^{6}+46656a^{5}b)\_Z^{6}+3888a^{4}\_Z^{4}+108a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{15552ia^{6}}{b}-15552ia^{5}\right)\_R^{5}+\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+3
*_Z^4*a+3*_Z^2*a+a+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^6 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.96, size = 15483, normalized size = 90.54

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^6),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a^4 + a^3*b) - 1/(a^2 + a*b)^2)/(
-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b)) - 1/46656/(a^2
 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/933
12/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b)) - 1/46656/(a^2 + a*b
)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 72/(a^2 + a*b))*log(-1/5184*(a^5 +
 a^4*b - 2*(a^5 + a^4*b)*cos(x)^2)*((-I*sqrt(3) + 1)*(1/(a^4 + a^3*b) - 1/(
a^2 + a*b)^2)/(-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b))
 - 1/46656/(a^2 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 1296*(I*sqrt(
3) + 1)*(-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 + a^3*b)*(a^2 + a*b)) - 1/4
6656/(a^2 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(1/3) - 72/(a^2 + a*b))^2 +
 (2*a + b)*cos(x)^2 + 1/15552*sqrt(1/2)*((a^6 + a^5*b)*((-I*sqrt(3) + 1)*(1
/(a^4 + a^3*b) - 1/(a^2 + a*b)^2)/(-1/93312/(a^6 + a^5*b) + 1/31104/((a^4 +
 a^3*b)*(a^2 + a*b)) - 1/46656/(a^2 + a*b)^3 + 1/93312*b/((a + b)^2*a^5))^(
1/3) - 1296*(I*sqrt(3) +  ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\cos^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**6),x)
```

```
[Out] Integral(1/(a + b*cos(x)**6), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^6),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.08, size = 184, normalized size = 1.08

$$\sum_{k=1}^{6}\ln\left(\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)^2\,a^3\,b^3\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)^2\,a^2\,36+1\right)\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)\,a\tan(x)\,6-1\right)\,36\right)\,\mathrm{root}\left(46656\,a^5\,b\,d^6+46656\,a^6\,d^6+3888\,a^4\,d^4+108\,a^2\,d^2+1,d,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^6),x)
```

```
[Out] symsum(log(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2
*d^2 + 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a
^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 + 46656*
a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(46656*a
^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

3.76 $\int \frac{1}{a+b\cos^8(x)}\,dx$

Optimal. Leaf size=245

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+$$

```
[Out] 1/4*arctan(cot(x)*((-a)^(1/4)-b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(
1/4)-b^(1/4))^(1/2)+1/4*arctan(cot(x)*((-a)^(1/4)-I*b^(1/4))^(1/2)/(-a)^(1/
8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)+1/4*arctan(cot(x)*((-a)^(1/4)+I
*b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)+1/4*arc
tan(cot(x)*((-a)^(1/4)+b^(1/4))^(1/2)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+b^
(1/4))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3290, 3260, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\cot(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^8)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Cot[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt
[(-a)^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Cot[x])/(-
a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[(-a)^(
1/4) + b^(1/4)]*Cot[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + b^(1/4)
]) + ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Cot[x])/(-a)^(5/8)]/(4*(-a)^(3/8)
*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\cos^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \cot(x)\right)}{4a} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \cot(x)\right)}{4a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{}}}{}\right)}{4(-a)^{7/8}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.29, size = 172, normalized size = 0.70

$$8\text{RootSum}\left[b+8b\#1+28b\#1^2+56b\#1^3+256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7+b\#1^8\&, \frac{2\text{ArcTan}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{b+7b\#1+21b\#1^2+128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 + 56*
b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #
1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 +
128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 76, normalized size = 0.31

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a+b\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8+16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{4194304ia^8}{b}+4\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*cos(x)^8 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665467 vs. $2(165) = 330$.
time = 6.45, size = 665467, normalized size = 2716.19

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^8),x, algorithm="fricas")
```

```
[Out] -1/384*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(((a^3*sqrt(-(2*a*b*sqrt(-b/a) + a*b
 - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + a^2*b*sqrt(-(2*a*b*sqrt(-
b/a) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sqrt(-b/a)
 - b)^2*a/((a^3 + a^2*b)^2*b) - 3*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a) + a*b -
b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + (2*a^3*sqrt(-(2*a*b*sqrt(-b/
a) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sqrt(-b/a) -
 b)/((a^5 + a^4*b)*sqrt(-b/a)))/(-1/1572864*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a
) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) + (2*a^3*sqrt(-(2*a*
b*sqrt(-b/a) + a*b - b^2)/((a^6 + 2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sq
rt(-b/a) - b)*((a^3*sqrt(-(2*a*b*sqrt(-b/a) + a*b - b^2)/((a^6 + 2*a^5*b +
a^4*b^2)*sqrt(-b/a))) + a^2*b*sqrt(-(2*a*b*sqrt(-b/a) + a*b - b^2)/((a^6 +
```

```
2*a^5*b + a^4*b^2)*sqrt(-b/a))) - 3*a)*sqrt(-b/a) - b)*a/((a^5 + a^4*b)*(a^
3 + a^2*b)*b) + 1/524288*(2*a^2*b*sqrt(-(2*a*b*sqrt(-b/a) + a*b - b^2)/((a^
6 + 2*a^5*b + a^4*b^2)*sq ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\cos^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**8),x)
```

```
[Out] Integral(1/(a + b*cos(x)**8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^8),x, algorithm="giac")
```

```
[Out] integrate(1/(b*cos(x)^8 + a), x)
```

Mupad [B]
time = 3.42, size = 216, normalized size = 0.88

$$\sum_{k=1}^{8}\ln\left(\mathrm{root}\left(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\right)^4\,a^5\,b^5\,\left(\mathrm{root}\left(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\right)^2\,a^2\,64+1\right)\,\left(\mathrm{root}\left(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\right)\,a\tan(x)\,8-1\right)\,4096\right)\,\mathrm{root}\left(16777216\,a^7\,b\,d^8+16777216\,a^8\,d^8+1048576\,a^6\,d^6+24576\,a^4\,d^4+256\,a^2\,d^2+1,d,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(x)^8),x)
```

```
[Out] symsum(log(4096*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^
6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^4*a^5*b^5*(64*root(16777216*a^7*
b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 +
1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576
*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 1))*root(16777
216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^
2*d^2 + 1, d, k), k, 1, 8)
```

3.77 $\int \frac{1}{a-b\cos^5(x)}\,dx$

Optimal. Leaf size=494

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+\cdots}}{\sqrt{\sqrt[5]{a}\cdots}}\right.}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{\cdots}}}$$

```
[Out] 2/5*arctan((a^(1/5)+b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-b^(1/5))^(1/2))/a^(4
/5)/(a^(1/5)-b^(1/5))^(1/2)/(a^(1/5)+b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1
)^(1/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2))/a^(4/
5)/(a^(1/5)-(-1)^(1/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(1/5)*b^(1/5))^(1/2)+2/
5*arctan((a^(1/5)+(-1)^(2/5)*b^(1/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(2/5)*
b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(2/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(2
/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)-(-1)^(3/5)*b^(1/5))^(1/2)*tan(1/2*x)
/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-(-1)^(3/5)*b^(1/5))^(
1/2)/(a^(1/5)+(-1)^(3/5)*b^(1/5))^(1/2)+2/5*arctan((a^(1/5)+(-1)^(4/5)*b^(1
/5))^(1/2)*tan(1/2*x)/(a^(1/5)-(-1)^(4/5)*b^(1/5))^(1/2))/a^(4/5)/(a^(1/5)-
(-1)^(4/5)*b^(1/5))^(1/2)/(a^(1/5)+(-1)^(4/5)*b^(1/5))^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3292, 2738, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{2/5}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{3/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-(-1)^{4/5}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+(-1)^{4/5}\sqrt[5]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a^(1/5) + b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - b^(1/5)]])/(5*a
^(4/5)*Sqrt[a^(1/5) - b^(1/5)]*Sqrt[a^(1/5) + b^(1/5)]) + (2*ArcTan[(Sqrt[a
^(1/5) - (-1)^(1/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(1/5)*b^(1/5)]])
/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(1/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(1/5)*b^
(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1
/5) - (-1)^(2/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(2/5)*b^(1/5)]*S
qrt[a^(1/5) + (-1)^(2/5)*b^(1/5)]) + (2*ArcTan[(Sqrt[a^(1/5) - (-1)^(3/5)*b
^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]])/(5*a^(4/5)*Sqrt[a^(1
/5) - (-1)^(3/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(3/5)*b^(1/5)]) + (2*ArcTan[(
Sqrt[a^(1/5) + (-1)^(4/5)*b^(1/5)]*Tan[x/2])/Sqrt[a^(1/5) - (-1)^(4/5)*b^(1
/5)]])/(5*a^(4/5)*Sqrt[a^(1/5) - (-1)^(4/5)*b^(1/5)]*Sqrt[a^(1/5) + (-1)^(4
/5)*b^(1/5)])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a-b\cos^5(x)}\,dx = \int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\cos(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)\right)}+\frac{}{5a^{4/5}\left(\sqrt[5]{a}-(\right.}\right.$$

$$=\frac{\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\cos(x)}\,dx}{5a^{4/5}}+\frac{\int \frac{}{\sqrt[5]{}}}{}$$

$$=\frac{2\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}+\left(\sqrt[5]{a}+\sqrt[5]{b}\right)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}+\left(\right.}\right.}{5}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{b}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\tan\left(\frac{x}{2}\right)}{\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{a}-\sqrt[5]{-1}\sqrt[5]{b}}\sqrt{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.19, size = 130, normalized size = 0.26

$$-\frac{8}{5}\text{RootSum}\left[b+5b\#1^2+10b\#1^4-32a\#1^5+10b\#1^6+5b\#1^8+b\#1^{10}\&,\frac{2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3}{b+4b\#1^2-16a\#1^3+6b\#1^4+4b\#1^6+b\#1^8}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Cos[x]^5)^(-1),x]
```

```
[Out] (-8*RootSum[b + 5*b*#1^2 + 10*b*#1^4 - 32*a*#1^5 + 10*b*#1^6 + 5*b*#1^8 + b
*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - I*Log[1 - 2*Cos[x]*#1 + #
1^2]*#1^3)/(b + 4*b*#1^2 - 16*a*#1^3 + 6*b*#1^4 + 4*b*#1^6 + b*#1^8) & ])/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 148, normalized size = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{10}+(5a-5b)\_Z^{8}+(10a+10b)\_Z^{6}+(10a-10b)\_Z^{4}+(5a+5b)\_Z^{2}+a-b\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_\right.}{\_R^{9}a+\_R^{9}b+4\_R^{7}a-4\_R^{7}b+6\_R^{5}a}\right.}{5}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(9765625a^{10}-9765625a^{8}b^{2})\_Z^{10}+1953125a^{8}\_Z^{8}+156250a^{6}\_Z^{6}+6250a^{4}\_Z^{4}+125a^{2}\_Z^{2}\right)}\_R\ln\left(e^{ix}+\left(\frac{11}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+_R^9*b+4*_R^7*a-4*_R^7*b+6*_R
^5*a+6*_R^5*b+4*_R^3*a-4*_R^3*b+_R*a+_R*b)*ln(tan(1/2*x)-_R),_R=RootOf((a+b
)*_Z^10+(5*a-5*b)*_Z^8+(10*a+10*b)*_Z^6+(10*a-10*b)*_Z^4+(5*a+5*b)*_Z^2+a-b
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^5),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^5 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\cos^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**5),x)
```

```
[Out] Integral(1/(a - b*cos(x)**5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*cos(x)^5 - a), x)
```

Mupad [B]
time = 7.83, size = 1518, normalized size = 3.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^5),x)
```

```
[Out] symsum(log(-(10995116277760*b^7*(a + b)*(56*root(9765625*a^8*b^2*d^10 - 976
5625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)*a - 7*cot(x/2) + root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^
10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d,
k)*b + 5800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^3 + 225000*ro
ot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*
d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 3875000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 25000000*root(9765625*a^8*b^2*d^10 -
 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*
a^2*d^2 - 1, d, k)^9*a^9 - 735*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1
0 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k
)^2*a^2*cot(x/2) - 28875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 19
53125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^
4*cot(x/2) - 503125*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125
*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^6*cot
```

```
(x/2) - 3281250*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^8*cot(x/2
) + 800*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b + 100000*root
(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^
6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b + 4000000*root(9765625*a^
8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^
4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b + 50000000*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^9*a^8*b + 125000*root(9765625*a^8*b^2*d^10 - 9765625*a
^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 -
1, d, k)^7*a^5*b^2 + 25000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^9*a^7*b^2 - 35*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^2*a*b*cot(x/2
) - 7000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^4*a^3*b*cot(x/2) - 3
50000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156
250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b*cot(x/2) - 5000
000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 15625
0*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b*cot(x/2) - 3125*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*cot(x/2) - 1718750*r
oot(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6
*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*cot(x/2)))/cot(x/2))
*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

3.78 $\int \frac{1}{a-b\cos^6(x)}\,dx$

Optimal. Leaf size=175

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}{\sqrt[6]{a}}\right.}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] -1/3*arctan(cot(x)*(a^(1/3)-b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/3
))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)/a^(1/6))/a^(5
/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)-1/3*arctan(cot(x)*(a^(1/3)-(-1)^(2/3
)*b^(1/3))^(1/2)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3290, 3260, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^6)^(-1),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Cot[x])/a^(1/6)]/(a^(5/6)*Sqrt[a^(1/3)
 - b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Cot[x])/a^(1/6)]/
(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) - ArcTan[(Sqrt[a^(1/3) - (-1
)^(2/3)*b^(1/3)]*Cot[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1
/3)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\cos^6(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{\sqrt[3]{-1}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\cos^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \cot(x)\right)}{3a} - \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \cot(x)\right)}{3a}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\cot(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{}}{}\right)}{3a^{5/6}\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.19, size = 146, normalized size = 0.83

$$-\frac{8}{3}\text{RootSum}\left[b+6b\#1+15b\#1^2-64a\#1^3+20b\#1^3+15b\#1^4+6b\#1^5+b\#1^6\&, \frac{2\text{ArcTan}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^2-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^2}{b+5b\#1-32a\#1^2+10b\#1^2+10b\#1^3+5b\#1^4+b\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Cos[x]^6)^(-1),x]
```

```
[Out] (-8*RootSum[b + 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 + 20*b*#1^3 + 15*b*#1^4 + 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*
Cos[2*x]*#1 + #1^2]*#1^2)/(b + 5*b*#1 - 32*a*#1^2 + 10*b*#1^2 + 10*b*#1^3 +
 5*b*#1^4 + b*#1^5) & ])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 62, normalized size = 0.35

method	result

default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3a\_Z^{4}+3a\_Z^{2}+a-b\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^{5}+2\_R^{3}+\_R}}{6a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(46656a^{6}-46656a^{5}b)\_Z^{6}+3888a^{4}\_Z^{4}+108a^{2}\_Z^{2}\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{15552ia^{6}}{b}-15552ia^{5}\right)\_R^{5}+\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a*sum((_R^4+2*_R^2+1)/(_R^5+2*_R^3+_R)*ln(tan(x)-_R),_R=RootOf(_Z^6*a+3
*_Z^4*a+3*_Z^2*a+a-b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^6),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^6 - a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.97, size = 16679, normalized size = 95.31

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^6),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a^4 - a^3*b) - 1/(a^2 - a*b)^2)/(-
1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) - 1/46656/(a^2
- a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9331
2/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) - 1/46656/(a^2 - a*b)
^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 72/(a^2 - a*b))*log(1/5184*(a^5 - a
^4*b - 2*(a^5 - a^4*b)*cos(x)^2)*((-I*sqrt(3) + 1)*(1/(a^4 - a^3*b) - 1/(a^
2 - a*b)^2)/(-1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) -
 1/46656/(a^2 - a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 1296*(I*sqrt(3)
 + 1)*(-1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a^3*b)*(a^2 - a*b)) - 1/466
56/(a^2 - a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/3) - 72/(a^2 - a*b))^2 - (
2*a - b)*cos(x)^2 + 1/15552*sqrt(1/2)*((a^6 - a^5*b)*((-I*sqrt(3) + 1)*(1/(
a^4 - a^3*b) - 1/(a^2 - a*b)^2)/(-1/93312/(a^6 - a^5*b) + 1/31104/((a^4 - a
^3*b)*(a^2 - a*b)) - 1/46656/(a^2 - a*b)^3 + 1/93312*b/((a - b)^2*a^5))^(1/
3) - 1296*(I*sqrt(3) + 1) ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a-b\cos^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**6),x)
```

```
[Out] Integral(1/(a - b*cos(x)**6), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^6),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.12, size = 184, normalized size = 1.05

$$\sum_{k=1}^{6}\ln\left(-\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\right)^2\,a^3\,b^3\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\right)^2\,a^2\,36+1\right)\,\left(\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\right)\,a\,\tan(x)\,6-1\right)\,36\right)\,\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^6),x)
```

```
[Out] symsum(log(-36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^
2*d^2 - 1, d, k)^2*a^3*b^3*(36*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*
a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2 + 1)*(6*root(46656*a^5*b*d^6 - 46656
*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root(46656*
a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k, 1, 6)
```

3.79 $\int \frac{1}{a-b\cos^8(x)}\,dx$

Optimal. Leaf size=213

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}-\frac{\text{A}}{}$$

```
[Out] -1/4*arctan(cot(x)*(a^(1/4)-b^(1/4))^(1/2)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/4
))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)-I*b^(1/4))^(1/2)/a^(1/8))/a^(7/8)/(a^(1
/4)-I*b^(1/4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)+I*b^(1/4))^(1/2)/a^(1/8))/a
^(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)-1/4*arctan(cot(x)*(a^(1/4)+b^(1/4))^(1/2)/
a^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3290, 3260, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}-\frac{\text{ArcTan}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Cos[x]^8)^(-1),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Cot[x])/a^(1/8)]/(a^(7/8)*Sqrt[a^(1/4)
 - b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Cot[x])/a^(1/8)]/(4*a^(7/8
)*Sqrt[a^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Cot[x])/a^
(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[a^(1/4) + b^(1/
4)]*Cot[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\cos^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}\,dx}{4a}+\frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}\,dx}{4a}+\frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}\,dx}{4a}+\frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\cos^2(x)}{\sqrt[4]{a}}}\,dx}{4a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx,x,\cot(x)\right)}{4a}-\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx,x,\cot(x)\right)}{4a}-$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\cot(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+}}{}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.23, size = 172, normalized size = 0.81

$$-8\text{RootSum}\left[b+8b\#1+28b\#1^2+56b\#1^3-256a\#1^4+70b\#1^4+56b\#1^5+28b\#1^6+8b\#1^7+b\#1^8\&,\frac{2\text{ArcTan}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{b+7b\#1+21b\#1^2-128a\#1^3+35b\#1^3+35b\#1^4+21b\#1^5+7b\#1^6+b\#1^7}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Cos[x]^8)^(-1),x]
[Out] -8*RootSum[b + 8*b*#1 + 28*b*#1^2 + 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 + 56
*b*#1^5 + 28*b*#1^6 + 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
#1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(b + 7*b*#1 + 21*b*#1^2 -
 128*a*#1^3 + 35*b*#1^3 + 35*b*#1^4 + 21*b*#1^5 + 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.46, size = 78, normalized size = 0.37

method	result

default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a-b\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8a}$
risch	$\sum_{\_R=\mathrm{RootOf}\left(1+(16777216a^8-16777216a^7b)\_Z^8+1048576a^6\_Z^6+24576a^4\_Z^4+256a^2\_Z^2\right)} \_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{4194304ia^8}{b}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*cos(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(_Z^8*a+4*_Z^6*a+6*_Z^4*a+4*_Z^2*a+a-b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^8),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*cos(x)^8 - a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643291 vs. $2(133) = 266$.
time = 6.61, size = 643291, normalized size = 3020.15

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^8),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(1/2)*sqrt(1/6)*sqrt((6*sqrt(1/2)*sqrt(1/6)*(a^2 - a*b)*sqrt(-((a^
5 - 2*a^4*b + a^3*b^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(((a^3*sqrt((2*a*b*s
qrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - a^2*b*sqrt((
2*a*b*sqrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - 3*a)*
sqrt(b/a) - b)^2*a/((a^3 - a^2*b)^2*b) + 3*(2*a^2*b*sqrt((2*a*b*sqrt(b/a) -
 a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) + (2*a^3*sqrt((2*a*b*sqr
t(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - 3*a)*sqrt(b/a)
 - b)/((a^5 - a^4*b)*sqrt(b/a)))/(9*(2*a^2*b*sqrt((2*a*b*sqrt(b/a) - a*b -
b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) + (2*a^3*sqrt((2*a*b*sqrt(b/a)
- a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a))) - 3*a)*sqrt(b/a) - b)*(
(a^3*sqrt((2*a*b*sqrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(b/a
))) - a^2*b*sqrt((2*a*b*sqrt(b/a) - a*b - b^2)/((a^6 - 2*a^5*b + a^4*b^2)*s
```

```
qrt(b/a))) - 3*a)*sqrt(b/a) - b)*a/((a^5 - a^4*b)*(a^3 - a^2*b)*b) + 27*(2*
a^2*b*sqrt((2*a*b*sqrt(b/ ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\cos^{8}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)**8),x)
```

```
[Out] Integral(1/(a - b*cos(x)**8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*cos(x)^8),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*cos(x)^8 - a), x)
```

Mupad [B]
time = 3.42, size = 216, normalized size = 1.01

$$\sum_{k=1}^{8} \ln\left(-\mathrm{root}\left(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\right)^4\,a^5\,b^5\,\left(\mathrm{root}\left(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\right)^2\,a^2\,64+1\right)\,\left(\mathrm{root}\left(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\right)\,a\,\tan(x)\,8-1\right)\,4096\right)\,\mathrm{root}\left(16777216\,a^7\,b\,d^8-16777216\,a^8\,d^8-1048576\,a^6\,d^6-24576\,a^4\,d^4-256\,a^2\,d^2-1,d,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*cos(x)^8),x)
```

```
[Out] symsum(log(-4096*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d
^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^4*a^5*b^5*(64*root(16777216*a^7
*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 -
 1, d, k)^2*a^2 + 1)*(8*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 104857
6*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 1))*root(1677
7216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a
^2*d^2 - 1, d, k), k, 1, 8)
```

3.80 $\int \frac{1}{1+\cos^5(x)}\,dx$

Optimal. Leaf size=223

$$\frac{2\text{ArcTan}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\text{ArcTan}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}}-$$

```
[Out] 1/5*sin(x)/(cos(x)+1)-2/5*arctanh(tan(1/2*x)/((-1+(-1)^(1/5))/(1+(-1)^(1/5)
))^(1/2))/(-1+(-1)^(2/5))^(1/2)+2/5*arctan(((1-(-1)^(4/5))/(1+(-1)^(4/5)))^
(1/2)*tan(1/2*x))/(1+(-1)^(3/5))^(1/2)-2/5*arctanh(((-1-(-1)^(3/5))/(1-(-1)
^(3/5)))^(1/2)*tan(1/2*x))*((-1-(-1)^(3/5))/(1-(-1)^(3/5)))^(1/2)/(1+(-1)^(
3/5))+2/5*arctan(((1-(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2)*tan(1/2*x))/(1-(-1)^
(4/5))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3292, 2727, 2738, 214, 211}

$$\frac{2\text{ArcTan}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\text{ArcTan}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{\sin(x)}{5(\cos(x)+1)}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}}\right)}{5\sqrt{(-1)^{2/5}-1}}-\frac{2\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tanh^{-1}\left(\sqrt{-\frac{1+(-1)^{3/5}}{1-(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\left(1+(-1)^{3/5}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[Sqrt[(1 - (-1)^(2/5))/(1 + (-1)^(2/5))]*Tan[x/2]])/(5*Sqrt[1 - (-
1)^(4/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(4/5))/(1 + (-1)^(4/5))]*Tan[x/2]])/(
5*Sqrt[1 + (-1)^(3/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(1/5))/(1 +
(-1)^(1/5)))]])/(5*Sqrt[-1 + (-1)^(2/5)]) - (2*Sqrt[-((1 + (-1)^(3/5))/(1 -
 (-1)^(3/5)))]*ArcTanh[Sqrt[-((1 + (-1)^(3/5))/(1 - (-1)^(3/5)))]*Tan[x/2]]
)/(5*(1 + (-1)^(3/5))) + Sin[x]/(5*(1 + Cos[x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\cos^5(x)}\,dx &= \int \left(-\frac{1}{5(-1-\cos(x))} - \frac{1}{5\left(-1+\sqrt[5]{-1}\,\cos(x)\right)} - \frac{1}{5\left(-1-(-1)^{2/5}\cos(x)\right)} - \frac{1}{5\left(-1+(\right.}\right.\\ &= -\left(\frac{1}{5}\int \frac{1}{-1-\cos(x)}\,dx\right) - \frac{1}{5}\int \frac{1}{-1+\sqrt[5]{-1}\,\cos(x)}\,dx - \frac{1}{5}\int \frac{1}{-1-(-1)^{2/5}\cos(x)}\,dx\\ &= \frac{\sin(x)}{5(1+\cos(x))} - \frac{2}{5}\text{Subst}\left(\int \frac{1}{-1+\sqrt[5]{-1}+\left(-1-\sqrt[5]{-1}\right)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{2}{5}\text{Subst}\\ &= \frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}\,\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}}\,\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}} - \frac{2\tanh^{-1}\left(\right.}{5\sqrt{-}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 378, normalized size = 1.70

$$-\frac{1}{10}\text{RootSum}\left[1-2\#1+8\#1^2-14\#1^3+30\#1^4-14\#1^5+8\#1^6-2\#1^7+\#1^8\&,\frac{2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)-8\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1+4i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1+30\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2-15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2-80\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3+40i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3+30\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4-15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4-8\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^5+4i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^5+2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{-1+8\#1-21\#1^2+60\#1^3-35\#1^4+24\#1^5-7\#1^6+4\#1^7}\&\right]+\frac{1}{5}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^5)^(-1),x]
```

```
[Out] -1/10*RootSum[1 - 2*#1 + 8*#1^2 - 14*#1^3 + 30*#1^4 - 14*#1^5 + 8*#1^6 - 2*
#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos[x]*#1 + #
1^2] - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + (4*I)*Log[1 - 2*Cos[x]*#1 + #1^2
]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 - 2*Cos[x]*#1 +
#1^2]*#1^2 - 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 + (40*I)*Log[1 - 2*Cos[x]
*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (15*I)*Log[1 - 2*
Cos[x]*#1 + #1^2]*#1^4 - 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^5 + (4*I)*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 - I*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^6)/(-1 + 8*#1 - 21*#1^2 + 60*#1^3 - 35*#1^4 + 24*#
1^5 - 7*#1^6 + 4*#1^7) & ] + Tan[x/2]/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 62, normalized size = 0.28

method	result
default	$\frac{\tan\left(\frac{x}{2}\right)}{5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^{8}+10\_Z^{4}+1\right)}\frac{\left(5\_R^{6}+5\_R^{4}+5\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{7}+\_R^{3}}\right)}{50}$
risch	$\frac{2i}{5(e^{ix}+1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^{8}+156250\_Z^{6}+6250\_Z^{4}+125\_Z^{2}+1\right)}\_R\ln\left(e^{ix}-2343750i\_R^{7}+234375\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*tan(1/2*x)+1/50*sum((5*_R^6+5*_R^4+5*_R^2+1)/(_R^7+_R^3)*ln(tan(1/2*x)-
_R),_R=RootOf(5*_Z^8+10*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^5),x, algorithm="maxima")
```

```
[Out] -1/5*(5*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(-2/5*((cos(7*x) - 4*
cos(6*x) + 15*cos(5*x) - 40*cos(4*x) + 15*cos(3*x) - 4*cos(2*x) + cos(x))*c
os(8*x) + (16*cos(6*x) - 44*cos(5*x) + 110*cos(4*x) - 44*cos(3*x) + 16*cos(
2*x) - 4*cos(x) + 1)*cos(7*x) - 2*cos(7*x)^2 + 4*(44*cos(5*x) - 110*cos(4*x
) + 44*cos(3*x) - 16*cos(2*x) + 4*cos(x) - 1)*cos(6*x) - 32*cos(6*x)^2 + (1
010*cos(4*x) - 420*cos(3*x) + 176*cos(2*x) - 44*cos(x) + 15)*cos(5*x) - 210
```

```
*cos(5*x)^2 + 10*(101*cos(3*x) - 44*cos(2*x) + 11*cos(x) - 4)*cos(4*x) - 12
00*cos(4*x)^2 + (176*cos(2*x) - 44*cos(x) + 15)*cos(3*x) - 210*cos(3*x)^2 +
 4*(4*cos(x) - 1)*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (sin(7*x) - 4*sin
(6*x) + 15*sin(5*x) - 40*sin(4*x) + 15*sin(3*x) - 4*sin(2*x) + sin(x))*sin(
8*x) + 2*(8*sin(6*x) - 22*sin(5*x) + 55*sin(4*x) - 22*sin(3*x) + 8*sin(2*x)
 - 2*sin(x))*sin(7*x) - 2*sin(7*x)^2 + 8*(22*sin(5*x) - 55*sin(4*x) + 22*si
n(3*x) - 8*sin(2*x) + 2*sin(x))*sin(6*x) - 32*sin(6*x)^2 + 2*(505*sin(4*x)
- 210*sin(3*x) + 88*sin(2*x) - 22*sin(x))*sin(5*x) - 210*sin(5*x)^2 + 10*(1
01*sin(3*x) - 44*sin(2*x) + 11*sin(x))*sin(4*x) - 1200*sin(4*x)^2 + 44*(4*s
in(2*x) - sin(x))*sin(3*x) - 210*sin(3*x)^2 - 32*sin(2*x)^2 + 16*sin(2*x)*s
in(x) - 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) - 8*cos(6*x) + 14*cos(5*x) - 30
*cos(4*x) + 14*cos(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(8*x) - cos(8*x)^2
+ 4*(8*cos(6*x) - 14*cos(5*x) + 30*cos(4*x) - 14*cos(3*x) + 8*cos(2*x) - 2*
cos(x) + 1)*cos(7*x) - 4*cos(7*x)^2 + 16*(14*cos(5*x) - 30*cos(4*x) + 14*co
s(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(6*x) - 64*cos(6*x)^2 + 28*(30*cos(4
*x) - 14*cos(3*x) + 8*cos(2*x) - 2*cos(x) + 1)*cos(5*x) - 196*cos(5*x)^2 +
60*(14*cos(3*x) - 8*cos(2*x) + 2*cos(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 28
*(8*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - 196*cos(3*x)^2 + 16*(2*cos(x) - 1)*
cos(2*x) - 64*cos(2*x)^2 - 4*cos(x)^2 + 4*(sin(7*x) - 4*sin(6*x) + 7*sin(5*
x) - 15*sin(4*x) + 7*sin(3*x) - 4*sin(2*x) + sin(x))*sin(8*x) - sin(8*x)^2
+ 8*(4*sin(6*x) - 7*sin(5*x) + 15*sin(4*x) - 7*sin(3*x) + 4*sin(2*x) - sin(
x))*sin(7*x) - 4*sin(7*x)^2 + 32*(7*sin(5*x) - 15*sin(4*x) + 7*sin(3*x) - 4
*sin(2*x) + sin(x))*sin(6*x) - 64*sin(6*x)^2 + 56*(15*sin(4*x) - 7*sin(3*x)
 + 4*sin(2*x) - sin(x))*sin(5*x) - 196*sin(5*x)^2 + 120*(7*sin(3*x) - 4*sin
(2*x) + sin(x))*sin(4*x) - 900*sin(4*x)^2 + 56*(4*sin(2*x) - sin(x))*sin(3*
x) - 196*sin(3*x)^2 - 64*sin(2*x)^2 + 32*sin(2*x)*sin(x) - 4*sin(x)^2 + 4*c
os(x) - 1), x) - 2*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**5),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.78, size = 535, normalized size = 2.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5 + 1),x)
```

```
[Out] tan(x/2)/5 + 2*atanh((603979776*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1
220703125 - (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5 - 1
)^(1/2))/1220703125 - 301989888/1220703125)) + (268435456*5^(1/2)*tan(x/2)*
(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1
/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/122070312
5 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 301989888/122070312
5)))*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((603979776*t
an(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((3355443
2*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5^(1/2))/122
0703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(1/2))/1220703125 + 301989888/122
0703125)) + (268435456*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1
/50)^(1/2))/(244140625*((33554432*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/1220
703125 + (134217728*5^(1/2))/1220703125 + (67108864*(- (2*5^(1/2))/5 - 1)^(
1/2))/1220703125 + 301989888/1220703125)))*((- (2*5^(1/2))/5 - 1)^(1/2)/50
- 1/50)^(1/2) - 2*atanh((603979776*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/12
20703125 - (134217728*5^(1/2))/1220703125 - (67108864*((2*5^(1/2))/5 - 1)^(
1/2))/1220703125 + 301989888/1220703125)) - (268435456*5^(1/2)*tan(x/2)*(-
((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*(
(2*5^(1/2))/5 - 1)^(1/2))/1220703125 - (134217728*5^(1/2))/1220703125 - (67
108864*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 + 301989888/1220703125)))*(- (
(2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) + 2*atanh((603979776*tan(x/2)*(((
2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140625*((33554432*5^(1/2)*((2
*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5^(1/2))/1220703125 - (6710
```

```
8864*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 301989888/1220703125)) - (2684
35456*5^(1/2)*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(244140
625*((33554432*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 + (134217728*5
^(1/2))/1220703125 - (67108864*((2*5^(1/2))/5 - 1)^(1/2))/1220703125 - 3019
89888/1220703125)))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

3.81 $\int \frac{1}{1+\cos^6(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] 1/6*arctan(1/2*2^(1/2)*tan(x))*2^(1/2)+1/3*arctan(tan(x)/(1-(-1)^(1/3))^(1/
2))/(1-(-1)^(1/3))^(1/2)+1/3*arctan(tan(x)/(1+(-1)^(2/3))^(1/2))/(1+(-1)^(2
/3))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3290, 3260, 209}

$$-\frac{\operatorname{ArcTan}\left(\sqrt{1-\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{\operatorname{ArcTan}\left(\sqrt{1+(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1+(-1)^{2/3}}}-\frac{\operatorname{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^6)^(-1),x]
```

```
[Out] x/(3*Sqrt[2]) - ArcTan[Sqrt[1 - (-1)^(1/3)]*Cot[x]]/(3*Sqrt[1 - (-1)^(1/3)]
) - ArcTan[Sqrt[1 + (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 + (-1)^(2/3)]) - ArcTan[(
Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
```

```
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\cos^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1+\cos^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\cos^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\cos^2(x)}\,dx$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(x)\right)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[3]{-1}\right)x^2}\,dx, x, \cot(x)\right.$$
$$= \frac{x}{3\sqrt{2}} - \frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1+(-1)^{2/3}}} - \frac{\tan^{-1}\left(\ldots\right.}{3\ldots}$$

Mathematica [A]
time = 0.18, size = 79, normalized size = 0.95

$$\frac{1}{12}\left(-2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\,\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)+2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2\tan(x)}{\sqrt{3}}\right)+\log(2-\sin(2x))-\log(2+\sin(2x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2
]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + Log[2 - Sin[2*x]] - Log[2 +
 Sin[2*x]])/12
```

Maple [A]
time = 0.12, size = 73, normalized size = 0.88

method	result
default	$\frac{\ln(\tan^2(x)-\tan(x)+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{6}-\frac{\ln(\tan^2(x)+\tan(x)+1)}{12}+\frac{\sqrt{3}\ldots}{}$
risch	$\frac{i\sqrt{2}\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{12}-\frac{i\sqrt{2}\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{12}+\frac{\ln\left(e^{2ix}-2i-i\sqrt{3}\right)}{12}+\frac{i\ln\left(e^{2ix}-2i-i\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(e^{2ix}\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))+1/6
*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)-1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*
arctan(1/3*(2*tan(x)+1)*3^(1/2))
```

Maxima [**A**]
time = 0.48, size = 72, normalized size = 0.87

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(x\right)+1\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(x\right)-1\right)\right)+\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(x\right)\right)-\frac{1}{12}\log\left(\tan\left(x\right)^{2}+\tan\left(x\right)+1\right)+\frac{1}{12}\log\left(\tan\left(x\right)^{2}-\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - 1/12*log(ta
n(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(58) = 116.
time = 0.48, size = 138, normalized size = 1.66

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos\left(x\right)\sin\left(x\right)+\sqrt{3}}{3\left(2\cos\left(x\right)^{2}-1\right)}\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos\left(x\right)\sin\left(x\right)-\sqrt{3}}{3\left(2\cos\left(x\right)^{2}-1\right)}\right)-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)-\frac{1}{24}\log\left(-\cos\left(x\right)^{4}+\cos\left(x\right)^{2}+2\cos\left(x\right)\sin\left(x\right)+1\right)+\frac{1}{24}\log\left(-\cos\left(x\right)^{4}+\cos\left(x\right)^{2}-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^6),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1
)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^
2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*si
n(x))) - 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/24*log(-c
os(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**6),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(58) = 116.
time = 0.46, size = 185, normalized size = 2.23

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)+\cos\left(2x\right)-2\sin\left(2x\right)+1}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)-\sin\left(2x\right)+2}\right)\right)+\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)-\cos\left(2x\right)-2\sin\left(2x\right)-1}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)+\sin\left(2x\right)+2}\right)\right)+\frac{1}{6}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-\cos\left(2x\right)+1}\right)\right)-\frac{1}{12}\log\left(\tan\left(x\right)^{2}+\tan\left(x\right)+1\right)+\frac{1}{12}\log\left(\tan\left(x\right)^{2}-\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(sq
rt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x +
arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x) +
sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt(2)*
sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) - 1/12*l
og(tan(x)^2 + tan(x) + 1) + 1/12*log(tan(x)^2 - tan(x) + 1)
```

Mupad [B]
time = 2.39, size = 99, normalized size = 1.19

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{6}+\mathrm{atan}\left(\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)-\mathrm{atan}\left(-\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)+\frac{(x-\mathrm{atan}(\tan(x)))\,\left(\frac{\pi\,\sqrt{2}}{6}+\pi\,\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)+\pi\,\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)\right)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6 + 1),x)
```

```
[Out] atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) - atan((tan(x)*
1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) + (2^(1/2)*atan((2^(1/2)*tan
(x))/2))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6) +
pi*(3^(1/2)/6 + 1i/6)))/pi
```

3.82 $\int \frac{1}{1+\cos^8(x)}\,dx$

Optimal. Leaf size=129

$$-\frac{\text{ArcTan}\left(\sqrt{1-\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\text{ArcTan}\left(\sqrt{1+\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\text{ArcTan}\left(\sqrt{1-(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\text{Ar}$$

```
[Out] -1/4*arctan(cot(x)*(1-(-1)^(1/4))^(1/2))/(1-(-1)^(1/4))^(1/2)-1/4*arctan(co
t(x)*(1+(-1)^(1/4))^(1/2))/(1+(-1)^(1/4))^(1/2)-1/4*arctan(cot(x)*(1-(-1)^(
3/4))^(1/2))/(1-(-1)^(3/4))^(1/2)-1/4*arctan(cot(x)*(1+(-1)^(3/4))^(1/2))/(
1+(-1)^(3/4))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3290, 3260, 209}

$$-\frac{\text{ArcTan}\left(\sqrt{1-\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}-\frac{\text{ArcTan}\left(\sqrt{1+\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}-\frac{\text{ArcTan}\left(\sqrt{1-(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1-(-1)^{3/4}}}-\frac{\text{ArcTan}\left(\sqrt{1+(-1)^{3/4}}\,\cot(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^8)^(-1),x]
```

```
[Out] -1/4*ArcTan[Sqrt[1 - (-1)^(1/4)]*Cot[x]]/Sqrt[1 - (-1)^(1/4)] - ArcTan[Sqrt
[1 + (-1)^(1/4)]*Cot[x]]/(4*Sqrt[1 + (-1)^(1/4)]) - ArcTan[Sqrt[1 - (-1)^(3
/4)]*Cot[x]]/(4*Sqrt[1 - (-1)^(3/4)]) - ArcTan[Sqrt[1 + (-1)^(3/4)]*Cot[x]]
/(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
```

```
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\cos^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\,\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\,\cos^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\cos^2(x)}\,dx +$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \cot(x)\right)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[4]{-1}\right)x^2}\,d\right.$$

$$= -\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\cot(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1-(-1)}\right.}{4\sqrt{1-(-1}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.16, size = 141, normalized size = 1.09

$$8\text{RootSum}\left[1+8\#1+28\#1^2+56\#1^3+326\#1^4+56\#1^5+28\#1^6+8\#1^7+\#1^8\&, \frac{2\text{ArcTan}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)\#1^3-i\log\left(1-2\cos(2x)\#1+\#1^2\right)\#1^3}{1+7\#1+21\#1^2+163\#1^3+35\#1^4+21\#1^5+7\#1^6+\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[1 + 8*#1 + 28*#1^2 + 56*#1^3 + 326*#1^4 + 56*#1^5 + 28*#1^6 + 8*#
1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos[2
*x]*#1 + #1^2]*#1^3)/(1 + 7*#1 + 21*#1^2 + 163*#1^3 + 35*#1^4 + 21*#1^5 + 7
*#1^6 + #1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.35, size = 67, normalized size = 0.52

method	result
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+2\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$
risch	$\left(\sum_{\_R=\text{RootOf}\left(8192\_Z^4+(128-128i)\_Z^2+1-i\right)}\_R\ln\left(e^{2ix}+(1024+1024i)\_R^3+(-128+128i)\_R^2+(1\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=Roo
tOf(_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(cos(x)^8 + 1), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**8),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^8),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**B**]
time = 3.11, size = 1025, normalized size = 7.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^8 + 1),x)
```

```
[Out] atan((tan(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(
1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (2^(1/2)*tan(
x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^
(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) - (tan(x)*(2*2^(1/2) - 3)
^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) -
 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1) + (2^(1/2)*tan(x)*(2*
2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*
(2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) + 1))*((2*2^(1
/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(x)*(- (2*2^(1/2) - 3)^(1/2
)/128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 - (
2*2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(- (2*2^(1/2) - 3)^(1/2)/128 -
1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 - (2*2^(1/2
) - 3)^(1/2) - 1) + (tan(x)*(2*2^(1/2) - 3)^(1/2)*(- (2*2^(1/2) - 3)^(1/2)/
128 - 1/128)^(1/2)*8i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 - (2*
2^(1/2) - 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(2*2^(1/2) - 3)^(1/2)*(- (2*2^(1/
2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2 + 2
^(1/2)/2 - (2*2^(1/2) - 3)^(1/2) - 1))*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128
)^(1/2)*2i + atan((tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8i)
/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2)
 + 1) + (2^(1/2)*tan(x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/(
(2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2) +
 1) + (tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/12
8)^(1/2)*8i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2^(1/2)/2 + (- 2*2^(1/2
) - 3)^(1/2) + 1) + (2^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2)
 - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 + 2
^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2) + 1))*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1
/128)^(1/2)*2i - atan((tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*8
i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/
2) - 1) + (2^(1/2)*tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*4i)/(
(2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 + (- 2*2^(1/2) - 3)^(1/2) -
 1) - (tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)
^(1/2)*8i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/2)/2 + (- 2*2^(1/2)
- 3)^(1/2) - 1) - (2^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3
)^(1/2)/128 - 1/128)^(1/2)*4i)/((2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2 - 2^(1/
2)/2 + (- 2*2^(1/2) - 3)^(1/2) - 1))*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^
(1/2)*2i
```

3.83 $\int \frac{1}{1-\cos^5(x)}\,dx$

Optimal. Leaf size=205

$$\frac{2\text{ArcTan}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\text{ArcTan}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{-1+(-1)^{4/5}}}$$

```
[Out] -1/5*sin(x)/(1-cos(x))+2/5*arctan(((1-(-1)^(3/5))/(1+(-1)^(3/5)))^(1/2)*tan
(1/2*x))/(1+(-1)^(1/5))^(1/2)+2/5*arctan(((1-(-1)^(1/5))/(1+(-1)^(1/5)))^(1
/2)*tan(1/2*x))/(1-(-1)^(2/5))^(1/2)+2/5*arctanh(((-1-(-1)^(4/5))/(1-(-1)^(
4/5)))^(1/2)*tan(1/2*x))/(-1-(-1)^(3/5))^(1/2)-2/5*arctanh(tan(1/2*x)/((-1+
(-1)^(2/5))/(1+(-1)^(2/5)))^(1/2))/(-1+(-1)^(4/5))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3292, 2727, 2738, 211, 214}

$$\frac{2\text{ArcTan}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\text{ArcTan}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{\sin(x)}{5(1-\cos(x))}-\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}}}\right)}{5\sqrt{(-1)^{4/5}-1}}+\frac{2\tanh^{-1}\left(\sqrt{-\frac{1+(-1)^{4/5}}{1-(-1)^{4/5}}}\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{-1-(-1)^{3/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[Sqrt[(1 - (-1)^(1/5))/(1 + (-1)^(1/5))]*Tan[x/2]])/(5*Sqrt[1 - (-
1)^(2/5)]) + (2*ArcTan[Sqrt[(1 - (-1)^(3/5))/(1 + (-1)^(3/5))]*Tan[x/2]])/(
5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTanh[Tan[x/2]/Sqrt[-((1 - (-1)^(2/5))/(1 +
(-1)^(2/5)))]])/(5*Sqrt[-1 + (-1)^(4/5)]) + (2*ArcTanh[Sqrt[-((1 + (-1)^(4/
5))/(1 - (-1)^(4/5)))]*Tan[x/2]])/(5*Sqrt[-1 - (-1)^(3/5)]) - Sin[x]/(5*(1
- Cos[x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3292

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-\cos^5(x)}\,dx &= \int \left(\frac{1}{5(1-\cos(x))}+\frac{1}{5\left(1+\sqrt[5]{-1}\,\cos(x)\right)}+\frac{1}{5\left(1-(-1)^{2/5}\cos(x)\right)}+\frac{1}{5\left(1+(-1)^{3/5}\cos\right.}\right. \\
&= \frac{1}{5}\int \frac{1}{1-\cos(x)}\,dx+\frac{1}{5}\int \frac{1}{1+\sqrt[5]{-1}\,\cos(x)}\,dx+\frac{1}{5}\int \frac{1}{1-(-1)^{2/5}\cos(x)}\,dx+\frac{1}{5}\int \frac{}{1+} \\
&= -\frac{\sin(x)}{5(1-\cos(x))}+\frac{2}{5}\text{Subst}\left(\int \frac{1}{1+\sqrt[5]{-1}+\left(1-\sqrt[5]{-1}\right)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)+\frac{2}{5}\text{Subst}\left(\right. \\
&= \frac{2\tan^{-1}\left(\sqrt{\frac{1-\sqrt[5]{-1}}{1+\sqrt[5]{-1}}}\,\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\sqrt{\frac{1-(-1)^{3/5}}{1+(-1)^{3/5}}}\,\tan\left(\frac{x}{2}\right)\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\right.}{5\sqrt{-}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 378, normalized size = 1.84

$$-\frac{1}{5}\cot\left(\frac{x}{2}\right)+\frac{1}{10}\text{RootSum}\left[1+2\#1+8\#1^2+14\#1^3+30\#1^4+14\#1^5+8\#1^6+2\#1^7+\#1^8\&,\frac{2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)+8\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-4i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1+30\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2-15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2+80\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^3-40i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^3+30\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4-15i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4+8\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^5-4i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^5+2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{1+8\#1+21\#1^2+60\#1^3+35\#1^4+24\#1^5+7\#1^6+4\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^5)^(-1),x]
```

```
[Out] -1/5*Cot[x/2] + RootSum[1 + 2*#1 + 8*#1^2 + 14*#1^3 + 30*#1^4 + 14*#1^5 + 8
*#1^6 + 2*#1^7 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos
[x]*#1 + #1^2] + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - (4*I)*Log[1 - 2*Cos[x]
*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[1 - 2*Co
s[x]*#1 + #1^2]*#1^2 + 80*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - (40*I)*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^3 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (15*I)*
Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + 8*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^5 - (4
*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6
- I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(1 + 8*#1 + 21*#1^2 + 60*#1^3 + 35*#1
^4 + 24*#1^5 + 7*#1^6 + 4*#1^7) & ]/10
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 62, normalized size = 0.30

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+10\_Z^4+5\right)}\frac{\left(\_R^6+5\_R^4+5\_R^2+5\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^7+5\_R^3}\right)}{10}-\frac{1}{5\tan\left(\frac{x}{2}\right)}$
risch	$-\frac{2i}{5(\mathrm{e}^{ix}-1)}+\left(\sum_{\_R=\mathrm{RootOf}\left(1953125\_Z^8+156250\_Z^6+6250\_Z^4+125\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{ix}+2343750i\_R^7-23437\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*sum((_R^6+5*_R^4+5*_R^2+5)/(_R^7+5*_R^3)*ln(tan(1/2*x)-_R),_R=RootOf(_
Z^8+10*_Z^4+5))-1/5/tan(1/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^5),x, algorithm="maxima")
```

```
[Out] 1/5*(5*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(2/5*((cos(7*x) + 4*co
s(6*x) + 15*cos(5*x) + 40*cos(4*x) + 15*cos(3*x) + 4*cos(2*x) + cos(x))*cos
(8*x) + (16*cos(6*x) + 44*cos(5*x) + 110*cos(4*x) + 44*cos(3*x) + 16*cos(2*
x) + 4*cos(x) + 1)*cos(7*x) + 2*cos(7*x)^2 + 4*(44*cos(5*x) + 110*cos(4*x)
+ 44*cos(3*x) + 16*cos(2*x) + 4*cos(x) + 1)*cos(6*x) + 32*cos(6*x)^2 + (101
0*cos(4*x) + 420*cos(3*x) + 176*cos(2*x) + 44*cos(x) + 15)*cos(5*x) + 210*c
os(5*x)^2 + 10*(101*cos(3*x) + 44*cos(2*x) + 11*cos(x) + 4)*cos(4*x) + 1200
```

```
*cos(4*x)^2 + (176*cos(2*x) + 44*cos(x) + 15)*cos(3*x) + 210*cos(3*x)^2 + 4
*(4*cos(x) + 1)*cos(2*x) + 32*cos(2*x)^2 + 2*cos(x)^2 + (sin(7*x) + 4*sin(6
*x) + 15*sin(5*x) + 40*sin(4*x) + 15*sin(3*x) + 4*sin(2*x) + sin(x))*sin(8*
x) + 2*(8*sin(6*x) + 22*sin(5*x) + 55*sin(4*x) + 22*sin(3*x) + 8*sin(2*x) +
 2*sin(x))*sin(7*x) + 2*sin(7*x)^2 + 8*(22*sin(5*x) + 55*sin(4*x) + 22*sin(
3*x) + 8*sin(2*x) + 2*sin(x))*sin(6*x) + 32*sin(6*x)^2 + 2*(505*sin(4*x) +
210*sin(3*x) + 88*sin(2*x) + 22*sin(x))*sin(5*x) + 210*sin(5*x)^2 + 10*(101
*sin(3*x) + 44*sin(2*x) + 11*sin(x))*sin(4*x) + 1200*sin(4*x)^2 + 44*(4*sin
(2*x) + sin(x))*sin(3*x) + 210*sin(3*x)^2 + 32*sin(2*x)^2 + 16*sin(2*x)*sin
(x) + 2*sin(x)^2 + cos(x))/(2*(2*cos(7*x) + 8*cos(6*x) + 14*cos(5*x) + 30*c
os(4*x) + 14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(8*x) + cos(8*x)^2 +
4*(8*cos(6*x) + 14*cos(5*x) + 30*cos(4*x) + 14*cos(3*x) + 8*cos(2*x) + 2*co
s(x) + 1)*cos(7*x) + 4*cos(7*x)^2 + 16*(14*cos(5*x) + 30*cos(4*x) + 14*cos(
3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(6*x) + 64*cos(6*x)^2 + 28*(30*cos(4*x
) + 14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(5*x) + 196*cos(5*x)^2 + 60
*(14*cos(3*x) + 8*cos(2*x) + 2*cos(x) + 1)*cos(4*x) + 900*cos(4*x)^2 + 28*(
8*cos(2*x) + 2*cos(x) + 1)*cos(3*x) + 196*cos(3*x)^2 + 16*(2*cos(x) + 1)*co
s(2*x) + 64*cos(2*x)^2 + 4*cos(x)^2 + 4*(sin(7*x) + 4*sin(6*x) + 7*sin(5*x)
 + 15*sin(4*x) + 7*sin(3*x) + 4*sin(2*x) + sin(x))*sin(8*x) + sin(8*x)^2 +
8*(4*sin(6*x) + 7*sin(5*x) + 15*sin(4*x) + 7*sin(3*x) + 4*sin(2*x) + sin(x)
)*sin(7*x) + 4*sin(7*x)^2 + 32*(7*sin(5*x) + 15*sin(4*x) + 7*sin(3*x) + 4*s
in(2*x) + sin(x))*sin(6*x) + 64*sin(6*x)^2 + 56*(15*sin(4*x) + 7*sin(3*x) +
 4*sin(2*x) + sin(x))*sin(5*x) + 196*sin(5*x)^2 + 120*(7*sin(3*x) + 4*sin(2
*x) + sin(x))*sin(4*x) + 900*sin(4*x)^2 + 56*(4*sin(2*x) + sin(x))*sin(3*x)
 + 196*sin(3*x)^2 + 64*sin(2*x)^2 + 32*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos
(x) + 1), x) - 2*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^5),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**5),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.45, size = 403, normalized size = 1.97

$$2\,\mathrm{atanh}\left(\frac{50\,\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\,\sqrt{5}\,\sqrt{-\frac{2\sqrt{5}}{5}-1}+2\,\sqrt{5}-10\,\sqrt{-\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-2\,\mathrm{atanh}\left(\frac{50\,\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-20\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\,\sqrt{5}\,\sqrt{-\frac{2\sqrt{5}}{5}-1}-2\,\sqrt{5}-10\,\sqrt{-\frac{2\sqrt{5}}{5}-1}+5}\right)\sqrt{-\frac{\sqrt{-\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-\frac{\cot\left(\frac{x}{2}\right)}{5}+2\,\mathrm{atanh}\left(\frac{50\,\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}+20\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\,\sqrt{5}\,\sqrt{\frac{2\sqrt{5}}{5}-1}-2\,\sqrt{5}+10\,\sqrt{\frac{2\sqrt{5}}{5}-1}-5}\right)\sqrt{-\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}-2\,\mathrm{atanh}\left(\frac{50\,\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}+20\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}}{5\,\sqrt{5}\,\sqrt{\frac{2\sqrt{5}}{5}-1}+2\,\sqrt{5}+10\,\sqrt{\frac{2\sqrt{5}}{5}-1}+5}\right)\sqrt{\frac{\sqrt{\frac{2\sqrt{5}}{5}-1}}{50}-\frac{1}{50}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^5 - 1),x)
```

```
[Out] 2*atanh((50*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^(
1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(-
(2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) - 5))
*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((50*tan(x/2)*(- (-
 (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 20*5^(1/2)*tan(x/2)*(- (- (2*5
^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/
2) - 2*5^(1/2) - 10*(- (2*5^(1/2))/5 - 1)^(1/2) + 5))*(- (- (2*5^(1/2))/5 -
 1)^(1/2)/50 - 1/50)^(1/2) - cot(x/2)/5 + 2*atanh((50*tan(x/2)*(- ((2*5^(1/
2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) + 20*5^(1/2)*tan(x/2)*(- ((2*5^(1/2))/5 -
 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2) - 2*5^(1/2
) + 10*((2*5^(1/2))/5 - 1)^(1/2) - 5))*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/
50)^(1/2) - 2*atanh((50*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
) + 20*5^(1/2)*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*5^(
1/2)*((2*5^(1/2))/5 - 1)^(1/2) + 2*5^(1/2) + 10*((2*5^(1/2))/5 - 1)^(1/2) +
 5))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

3.84 $\int \frac{1}{1-\cos^6(x)} dx$

Optimal. Leaf size=71

$$-\frac{\text{ArcTan}\left(\sqrt{1+\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\text{ArcTan}\left(\sqrt{1-(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{\cot(x)}{3}$$

```
[Out] -1/3*cot(x)-1/3*arctan(cot(x)*(1+(-1)^(1/3))^(1/2))/(1+(-1)^(1/3))^(1/2)-1/
3*arctan(cot(x)*(1-(-1)^(2/3))^(1/2))/(1-(-1)^(2/3))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3290, 3260, 209, 3254, 3852, 8}

$$-\frac{\text{ArcTan}\left(\sqrt{1+\sqrt[3]{-1}}\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}-\frac{\text{ArcTan}\left(\sqrt{1-(-1)^{2/3}}\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{\cot(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^6)^(-1),x]
```

```
[Out] -1/3*ArcTan[Sqrt[1 + (-1)^(1/3)]*Cot[x]]/Sqrt[1 + (-1)^(1/3)] - ArcTan[Sqrt
[1 - (-1)^(2/3)]*Cot[x]]/(3*Sqrt[1 - (-1)^(2/3)]) - Cot[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1-\cos^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\,\cos^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\cos^2(x)}\,dx$$

$$= \frac{1}{3}\int \csc^2(x)\,dx - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[3]{-1}\right)x^2}\,dx, x, \cot(x)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{}{1+(1}\right.$$

$$= -\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}} - \frac{1}{3}\text{Subst}\left(\int 1\,dx,\right.$$

$$= -\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\cot(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} - \frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\cot(x)\right)}{3\sqrt{1-(-1)^{2/3}}} - \frac{\cot(x)}{3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 117, normalized size = 1.65

$$\frac{(15+8\cos(2x)+\cos(4x))\sin(x)\left(6\cos(x)+i\sqrt[4]{-3}\left(3i+\sqrt{3}\right)\text{ArcTan}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(-i+\sqrt{3}\right)\tan(x)\right)\sin(x)+\sqrt[4]{-3}\left(-3i+\sqrt{3}\right)\text{ArcTan}\left(\frac{(-1)^{3/4}\left(i+\sqrt{3}\right)\tan(x)}{2\sqrt[4]{3}}\right)\sin(x)\right)}{144\left(-1+\cos^6(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^6)^(-1),x]
```

```
[Out] ((15 + 8*Cos[2*x] + Cos[4*x])*Sin[x]*(6*Cos[x] + I*(-3)^(1/4)*(3*I + Sqrt[3
])*ArcTan[((-1/3)^(1/4)*(-I + Sqrt[3])*Tan[x])/2]*Sin[x] + (-3)^(1/4)*(-3*I
 + Sqrt[3])*ArcTan[((-1)^(3/4)*(I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Sin[x]))/
(144*(-1 + Cos[x]^6))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(49) = 98$.
time = 0.17, size = 171, normalized size = 2.41

method	result
risch	$-\frac{2i}{3(e^{2ix}-1)}+\left(\sum_{\_R=\mathrm{RootOf}(3888\_Z^4+108\_Z^2+1)}\_R\ln\left(e^{2ix}+1296i\_R^3-216\_R^2-1\right)\right)$
default	$-\frac{1}{3\tan(x)}+\frac{\sqrt{3}\left(\frac{\sqrt{2\sqrt{3}-3}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{2\sqrt{3}-3}+\sqrt{3}\right)}{2}+\frac{2\left(\sqrt{3}+\frac{3}{2}\right)\arctan\left(\frac{2\tan(x)-\sqrt{2\sqrt{3}-3}}{\sqrt{2\sqrt{3}+3}}\right)}{\sqrt{2\sqrt{3}+3}}\right.}{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^6),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/tan(x)+1/18*3^(1/2)*(1/2*(2*3^(1/2)-3)^(1/2)*ln(tan(x)^2-tan(x)*(2*3^(
1/2)-3)^(1/2)+3^(1/2))+2*(3^(1/2)+3/2)/(2*3^(1/2)+3)^(1/2)*arctan((2*tan(x)
-(2*3^(1/2)-3)^(1/2))/(2*3^(1/2)+3)^(1/2)))-1/18*3^(1/2)*(1/2*(2*3^(1/2)-3)
^(1/2)*ln(tan(x)^2+tan(x)*(2*3^(1/2)-3)^(1/2)+3^(1/2))+2*(-3^(1/2)-3/2)/(2*
3^(1/2)+3)^(1/2)*arctan((2*tan(x)+(2*3^(1/2)-3)^(1/2))/(2*3^(1/2)+3)^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^6),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(1/3*((cos(3*x)
+ 4*cos(2*x) + cos(x))*cos(4*x) + (14*cos(2*x) + 4*cos(x) + 1)*cos(3*x) + 2
*cos(3*x)^2 + 2*(7*cos(x) + 2)*cos(2*x) + 24*cos(2*x)^2 + 2*cos(x)^2 + (sin
(3*x) + 4*sin(2*x) + sin(x))*sin(4*x) + 2*(7*sin(2*x) + 2*sin(x))*sin(3*x)
+ 2*sin(3*x)^2 + 24*sin(2*x)^2 + 14*sin(2*x)*sin(x) + 2*sin(x)^2 + cos(x))/
(2*(2*cos(3*x) + 6*cos(2*x) + 2*cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 4*(6*co
s(2*x) + 2*cos(x) + 1)*cos(3*x) + 4*cos(3*x)^2 + 12*(2*cos(x) + 1)*cos(2*x)
 + 36*cos(2*x)^2 + 4*cos(x)^2 + 4*(sin(3*x) + 3*sin(2*x) + sin(x))*sin(4*x)
 + sin(4*x)^2 + 8*(3*sin(2*x) + sin(x))*sin(3*x) + 4*sin(3*x)^2 + 36*sin(2*
x)^2 + 24*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1), x) - 3*(cos(2*x)^2
+ sin(2*x)^2 - 2*cos(2*x) + 1)*integrate(-1/3*((cos(3*x) - 4*cos(2*x) + cos
(x))*cos(4*x) + (14*cos(2*x) - 4*cos(x) + 1)*cos(3*x) - 2*cos(3*x)^2 + 2*(7
*cos(x) - 2)*cos(2*x) - 24*cos(2*x)^2 - 2*cos(x)^2 + (sin(3*x) - 4*sin(2*x)
```

```
 + sin(x))*sin(4*x) + 2*(7*sin(2*x) - 2*sin(x))*sin(3*x) - 2*sin(3*x)^2 - 2
4*sin(2*x)^2 + 14*sin(2*x)*sin(x) - 2*sin(x)^2 + cos(x))/(2*(2*cos(3*x) - 6
*cos(2*x) + 2*cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 4*(6*cos(2*x) - 2*cos(x)
+ 1)*cos(3*x) - 4*cos(3*x)^2 + 12*(2*cos(x) - 1)*cos(2*x) - 36*cos(2*x)^2 -
 4*cos(x)^2 + 4*(sin(3*x) - 3*sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 8*
(3*sin(2*x) - sin(x))*sin(3*x) - 4*sin(3*x)^2 - 36*sin(2*x)^2 + 24*sin(2*x)
*sin(x) - 4*sin(x)^2 + 4*cos(x) - 1), x) - 2*sin(2*x))/(cos(2*x)^2 + sin(2*
x)^2 - 2*cos(2*x) + 1)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(66) = 132$.
time = 9.01, size = 728, normalized size = 10.25

$$\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{36}+\frac{\sqrt{2}\cdot\sqrt[4]{3}\left(\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{12}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{36}+\frac{\sqrt{2}\cdot\sqrt[4]{3}\left(\operatorname{atan}\left(\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{12}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{36}+\frac{\sqrt{2}\cdot\sqrt[4]{3}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{12}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{36}+\frac{\sqrt{2}\cdot\sqrt[4]{3}\left(\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)}{3}+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{12}-\frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tan^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{24}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tan^{2}\left(\frac{x}{2}\right)-4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{72}-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(4\tan^{2}\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{72}+\frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(4\tan^{2}\left(\frac{x}{2}\right)+4\sqrt{2}\cdot\sqrt[4]{3}\tan\left(\frac{x}{2}\right)+4\sqrt{3}\right)}{24}-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tan^{2}\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{72}+\frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tan^{2}\left(\frac{x}{2}\right)-12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{24}-\frac{\sqrt{2}\cdot\sqrt[4]{3}\log\left(36\tan^{2}\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{24}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(36\tan^{2}\left(\frac{x}{2}\right)+12\sqrt{2}\cdot 3^{\frac{3}{4}}\tan\left(\frac{x}{2}\right)+12\sqrt{3}\right)}{72}+\frac{\tan\left(\frac{x}{2}\right)}{6}-\frac{1}{6\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**6),x)
```

```
[Out] sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*floor((x/2 - pi/
2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2) - 1) + pi*flo
or((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(1/4)*tan(x/2)
+ 1) + pi*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(1
/4)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/12 + sqrt(2)*3**(3/4)*(atan(
sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi/2)/pi))/36 + sqrt(2)*
3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 - 1) + pi*floor((x/2 - pi/2)/pi)
)/12 + sqrt(2)*3**(3/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 + 1) + pi*floor((
x/2 - pi/2)/pi))/36 + sqrt(2)*3**(1/4)*(atan(sqrt(2)*3**(3/4)*tan(x/2)/3 +
1) + pi*floor((x/2 - pi/2)/pi))/12 - sqrt(2)*3**(1/4)*log(4*tan(x/2)**2 - 4
*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/24 + sqrt(2)*3**(3/4)*log(4*tan(x/2
)**2 - 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 - sqrt(2)*3**(3/4)*log(4
*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/72 + sqrt(2)*3**(1/
4)*log(4*tan(x/2)**2 + 4*sqrt(2)*3**(1/4)*tan(x/2) + 4*sqrt(3))/24 - sqrt(2
)*3**(3/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2) + 12*sqrt(3))/
72 + sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 - 12*sqrt(2)*3**(3/4)*tan(x/2) + 1
2*sqrt(3))/24 - sqrt(2)*3**(1/4)*log(36*tan(x/2)**2 + 12*sqrt(2)*3**(3/4)*t
```

```
an(x/2) + 12*sqrt(3))/24 + sqrt(2)*3**(3/4)*log(36*tan(x/2)**2 + 12*sqrt(2)
*3**(3/4)*tan(x/2) + 12*sqrt(3))/72 + tan(x/2)/6 - 1/(6*tan(x/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(49) = 98$.
time = 0.49, size = 199, normalized size = 2.80

$$\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-\arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan\left(x\right)\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}\right)\right)\sqrt{6\sqrt{3}+9}+\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{3^{\frac{3}{4}}\left(3^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)-4\tan\left(x\right)\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}\right)\right)\sqrt{6\sqrt{3}+9}-\frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(\frac{1}{2}\left(\sqrt{6}\,3^{\frac{1}{4}}-3^{\frac{1}{4}}\sqrt{2}\right)\tan\left(x\right)+\tan\left(x\right)^{2}+\sqrt{3}\right)+\frac{1}{36}\sqrt{6\sqrt{3}-9}\log\left(-\frac{1}{2}\left(\sqrt{6}\,3^{\frac{1}{4}}-3^{\frac{1}{4}}\sqrt{2}\right)\tan\left(x\right)+\tan\left(x\right)^{2}+\sqrt{3}\right)-\frac{1}{3\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^6),x, algorithm="giac")
```

```
[Out] 1/18*(pi*floor(x/pi + 1/2) - arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(2
)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi*floor(x
/pi + 1/2) + arctan(-1/3*3^(3/4)*(3^(1/4)*(sqrt(6) - sqrt(2)) - 4*tan(x))/(
sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) - 1/36*sqrt(6*sqrt(3) - 9)*log(1/2
*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x) + tan(x)^2 + sqrt(3)) + 1/36*sq
rt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))*tan(x) + tan
(x)^2 + sqrt(3)) - 1/3/tan(x)
```

Mupad [B]
time = 2.29, size = 95, normalized size = 1.34

$$-\frac{1}{3\tan\left(x\right)}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\,\sqrt{6}\,\tan\left(x\right)\,\left(\frac{1}{27}-\frac{1}{27}\mathrm{i}\right)}{-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\,\left(3^{1/4}\,\left(1+1\mathrm{i}\right)+3^{3/4}\,\left(-1+1\mathrm{i}\right)\right)\,1\mathrm{i}}{36}+\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{3^{1/4}\,\sqrt{6}\,\tan\left(x\right)\,\left(\frac{1}{27}+\frac{1}{27}\mathrm{i}\right)}{\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}}\right)\,\left(3^{1/4}\,\left(1-\mathrm{i}\right)+3^{3/4}\,\left(-1-\mathrm{i}\right)\right)\,1\mathrm{i}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^6 - 1),x)
```

```
[Out] (6^(1/2)*atan((3^(1/4)*6^(1/2)*tan(x)*(1/27 - 1i/27))/((3^(1/2)*1i)/9 - 1/9
))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 - 1/(3*tan(x)) + (6^(1/2)*a
tan((3^(1/4)*6^(1/2)*tan(x)*(1/27 + 1i/27))/((3^(1/2)*1i)/9 + 1/9))*(3^(1/4
)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

3.85 $\int \frac{1}{1-\cos^8(x)}\,dx$

Optimal. Leaf size=89

$$\frac{x}{4\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{1-i}\,\cot(x)\right)}{4\sqrt{1-i}}-\frac{\text{ArcTan}\left(\sqrt{1+i}\,\cot(x)\right)}{4\sqrt{1+i}}-\frac{\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\cot(x)}{4}$$

```
[Out] -1/4*cot(x)-1/4*arctan(cot(x)*(1-I)^(1/2))/(1-I)^(1/2)-1/4*arctan(cot(x)*(1
+I)^(1/2))/(1+I)^(1/2)+1/8*x*2^(1/2)-1/8*arctan(cos(x)*sin(x)/(1+cos(x)^2+2
^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3290, 3260, 209, 3254, 3852, 8}

$$-\frac{\text{ArcTan}\left(\sqrt{1-i}\,\cot(x)\right)}{4\sqrt{1-i}}-\frac{\text{ArcTan}\left(\sqrt{1+i}\,\cot(x)\right)}{4\sqrt{1+i}}-\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}+\frac{x}{4\sqrt{2}}-\frac{\cot(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^8)^(-1),x]
```

```
[Out] x/(4*Sqrt[2]) - ArcTan[Sqrt[1 - I]*Cot[x]]/(4*Sqrt[1 - I]) - ArcTan[Sqrt[1
+ I]*Cot[x]]/(4*Sqrt[1 + I]) - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]
^2)]/(4*Sqrt[2]) - Cot[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3290

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^(4*(k/n))*Rt[-a/b, n/
2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cos^8(x)}\,dx &= \frac{1}{4}\int \frac{1}{1-\cos^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1-i\cos^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+i\cos^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+\cos^2(x)}\,d\\ &= \frac{1}{4}\int \csc^2(x)\,dx-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1-i)x^2}\,dx,x,\cot(x)\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1+i)x^2}\,d\right.\\ &= \frac{x}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{1-i}\,\cot(x)\right)}{4\sqrt{1-i}}-\frac{\tan^{-1}\left(\sqrt{1+i}\,\cot(x)\right)}{4\sqrt{1+i}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\\ &= \frac{x}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{1-i}\,\cot(x)\right)}{4\sqrt{1-i}}-\frac{\tan^{-1}\left(\sqrt{1+i}\,\cot(x)\right)}{4\sqrt{1+i}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\end{aligned}$$

Mathematica [A]
time = 0.17, size = 64, normalized size = 0.72

$$\frac{1}{8}\left(\frac{2\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{1-i}}\right)}{\sqrt{1-i}}+\frac{2\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{1+i}}\right)}{\sqrt{1+i}}+\sqrt{2}\,\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)-2\cot(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTan[Tan[x]/Sqrt[1 - I]])/Sqrt[1 - I] + (2*ArcTan[Tan[x]/Sqrt[1 + I]]
)/Sqrt[1 + I] + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]] - 2*Cot[x])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(65) = 130$.
time = 0.21, size = 186, normalized size = 2.09

method	result
risch	$-\frac{i}{2(e^{2ix}-1)} + \frac{\sqrt{-2+2i}\,\ln\left(e^{2ix}-i\sqrt{-2+2i}-\sqrt{-2+2i}+1-2i\right)}{16} - \frac{\sqrt{-2+2i}\,\ln\left(e^{2ix}+i\sqrt{-2+2i}+ \right.}{16}$
default	$-\frac{1}{4\tan(x)} + \frac{\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)\sqrt{2}}{8} - \frac{\sqrt{2}\left(-\frac{\sqrt{-2+2\sqrt{2}}\,\ln\left(\tan^2(x)-\tan(x)\sqrt{-2+2\sqrt{2}}+\sqrt{2}\right)}{2}+\frac{2(-}{}\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)^8),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/tan(x)+1/8*arctan(1/2*tan(x)*2^(1/2))*2^(1/2)-1/16*2^(1/2)*(-1/2*(-2+2
*2^(1/2))^(1/2)*ln(tan(x)^2-tan(x)*(-2+2*2^(1/2))^(1/2)+2^(1/2))+2*(-1-2^(1
/2))/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)-(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+
2)^(1/2)))-1/16*2^(1/2)*(1/2*(-2+2*2^(1/2))^(1/2)*ln(tan(x)^2+tan(x)*(-2+2*
2^(1/2))^(1/2)+2^(1/2))+2*(-1-2^(1/2))/(2*2^(1/2)+2)^(1/2)*arctan((2*tan(x)
+(-2+2*2^(1/2))^(1/2))/(2*2^(1/2)+2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^8),x, algorithm="maxima")
```

```
[Out] 1/8*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*arctan2(4*sqrt(2)*sin(2*x)/
(2*(2*sqrt(2) + 3)*cos(2*x) + cos(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17), (
cos(2*x)^2 + sin(2*x)^2 + 6*cos(2*x) + 1)/(2*(2*sqrt(2) + 3)*cos(2*x) + cos
(2*x)^2 + sin(2*x)^2 + 12*sqrt(2) + 17)) + 64*(sqrt(2)*cos(2*x)^2 + sqrt(2)
*sin(2*x)^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))*integrate(((4*cos(2*x) + 1)*cos
(4*x) + cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) + 22*cos(4*x)^2 + sin(8*x)*
sin(4*x) + 4*sin(6*x)*sin(4*x) + 22*sin(4*x)^2 + 4*sin(4*x)*sin(2*x))/(2*(4
*cos(6*x) + 22*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(22*cos
(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 44*(4*cos(2*x) + 1)*cos(
4*x) + 484*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 11*sin(4*x) + 2*sin
(2*x))*sin(8*x) + sin(8*x)^2 + 16*(11*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*
sin(6*x)^2 + 484*sin(4*x)^2 + 176*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos
(2*x) + 1), x) - 4*sqrt(2)*sin(2*x))/(sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)
^2 - 2*sqrt(2)*cos(2*x) + sqrt(2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3963 vs. $2(57) = 114$.
time = 18.29, size = 3963, normalized size = 44.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^8),x, algorithm="fricas")
```

```
[Out] -1/64*(2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(1/4*(32*(s
qrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2)
 + 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(
2) - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(
x)^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2
)*(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1)
 - 2*sqrt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*s
qrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2)
+ 5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58)
)*cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos
(x)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7
 + 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2
^(3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))
*sqrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8
*sqrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(
x)^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqr
t(2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2)
- 46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2)
 + 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*
sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(
2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2)
- 39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23)
- 2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^
(1/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(s
qrt(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*s
in(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 + 4
*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*s
qrt(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 2
56*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)
) - 2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqr
t(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) +
 22) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2)
 - 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)
^10 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*
(49*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) -
 2*sqrt(2) - 5)*cos(x)^4 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqr
```

```
t(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) +
5))*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2^(1/4)*(19*sqrt(2) + 58))*
cos(x)^11 + 4*(6*2^(3/4)*(3*sqrt(2) - 4) - 2^(1/4)*(19*sqrt(2) - 32))*cos(x
)^9 - 2*(2^(3/4)*(28*sqrt(2) - 27) - 4*2^(1/4)*(15*sqrt(2) - 2))*cos(x)^7 +
 2*(2^(3/4)*(9*sqrt(2) - 8) - 2*2^(1/4)*(15*sqrt(2) + 2))*cos(x)^5 - (2*2^(
3/4)*(sqrt(2) - 1) - 2^(1/4)*(13*sqrt(2) + 2))*cos(x)^3 - 2^(3/4)*cos(x))*s
qrt(2*sqrt(2) + 4)*sin(x) + 4*cos(x)^2 - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*s
qrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)
^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(
2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) -
46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) +
 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sq
rt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2)
 - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) -
39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) -
2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1
/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqr
t(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin
(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 16*(sqrt(2) - 1)*cos(x)^2 + 4*(
2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqr
t(2) + 4)*sin(x) + 4))/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256
*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))
+ 2*(2^(1/4)*cos(x)^2 - 2^(1/4))*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(
2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(19*sqrt(2) + 2
2) - 8*sqrt(2) - 52)*cos(x)^14 + 32*(sqrt(2)*(8*sqrt(2) + 19) + 2*sqrt(2) -
 37)*cos(x)^12 + 16*(2*sqrt(2)*(4*sqrt(2) - 13) - 22*sqrt(2) + 39)*cos(x)^1
0 - 8*(sqrt(2)*(41*sqrt(2) - 10) - 42*sqrt(2) - 2)*cos(x)^8 + 4*(sqrt(2)*(4
9*sqrt(2) + 6) - 32*sqrt(2) - 32)*cos(x)^6 - 8*(sqrt(2)*(6*sqrt(2) + 1) - 2
*sqrt(2) - 5)*cos(x)^4 - 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(
2) + 2))*cos(x)^15 - 8*(2^(3/4)*(3*sqrt(2) + 2) - 4*2^(1/4)*(4*sqrt(2) + 5)
)*cos(x)^13 - 4*(2*2^(3/4)*(3*sqrt(2) - 10) + 2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)**8),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(57) = 114.
time = 0.65, size = 222, normalized size = 2.49

$$\frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}-2\tan(x)\right)}{2\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}-\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^{2}+2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)+\frac{1}{16}\sqrt{\sqrt{2}-1}\log\left(\tan(x)^{2}-2^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}\tan(x)+\sqrt{2}\right)-\frac{1}{4\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)^8),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(1/2*2^(3/4)*
(2^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) +
1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(-sqrt(2)
 + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) - 1/16*sqrt(sqrt(2)
 - 1)*log(tan(x)^2 + 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2)) + 1/16*sq
rt(sqrt(2) - 1)*log(tan(x)^2 - 2^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(2))
 - 1/4/tan(x)
```

Mupad [B]
time = 2.27, size = 241, normalized size = 2.71

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{8}-\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)}+\frac{\sqrt{2}\,\tan(x)\,\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}-\frac{1}{16}\right)}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}+\frac{1}{16}\right)}-\frac{\sqrt{2}\,\tan(x)\,\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,1\mathrm{i}}{2\left(16\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}+\frac{1}{16}\right)}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right)-\frac{1}{4\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^8 - 1),x)
```

```
[Out] atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256
- 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) + 1/16)) - (2^(1/2)*tan(x)*(2^
(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/2)/
256 - 1/256)^(1/2) + 1/16)))*((- 2^(1/2)/256 - 1/256)^(1/2)*2i + (2^(1/2)/2
56 - 1/256)^(1/2)*2i) - atan((2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*
1i)/(2*(16*(2^(1/2)/256 - 1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) - 1/16
)) + (2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*1i)/(2*(16*(2^(1/2)/256 -
1/256)^(1/2)*(- 2^(1/2)/256 - 1/256)^(1/2) - 1/16)))*((- 2^(1/2)/256 - 1/25
6)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) - 1/(4*tan(x)) + (2^(1/2)*ata
n((2^(1/2)*tan(x))/2))/8
```

3.86 $\int \frac{\tan(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=17

$$-\log(\cos(x))+\frac{1}{2}\log\left(1+\cos^2(x)\right)$$

```
[Out] -ln(cos(x))+1/2*ln(1+cos(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3273, 36, 29, 31}

$$\frac{1}{2}\log\left(\cos^2(x)+1\right)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(1 + Cos[x]^2),x]
```

```
[Out] -Log[Cos[x]] + Log[1 + Cos[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{1+\cos^2(x)}\,dx = -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right)\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos^2(x)\right)$$
$$= -\log(\cos(x)) + \frac{1}{2}\log\left(1+\cos^2(x)\right)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\log(\cos(x)) + \frac{1}{2}\log\left(1+\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(1 + Cos[x]^2),x]
[Out] -Log[Cos[x]] + Log[1 + Cos[x]^2]/2
```

Maple [A]
time = 0.08, size = 16, normalized size = 0.94

method	result	size
derivativedivides	$-\ln\left(\cos\left(x\right)\right)+\frac{\ln\left(1+\cos^{2}\left(x\right)\right)}{2}$	16
default	$-\ln\left(\cos\left(x\right)\right)+\frac{\ln\left(1+\cos^{2}\left(x\right)\right)}{2}$	16
risch	$-\ln\left(e^{2ix}+1\right)+\frac{\ln\left(e^{4ix}+6\,e^{2ix}+1\right)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -ln(cos(x))+1/2*ln(1+cos(x)^2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 1.12

$$-\frac{1}{2}\log\left(\sin\left(x\right)^{2}-1\right)+\frac{1}{2}\log\left(\sin\left(x\right)^{2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2),x, algorithm="maxima")
[Out] -1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 2)
```

Fricas [A]
time = 0.54, size = 19, normalized size = 1.12

$$\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(x\right)^{2}+\frac{1}{2}\right)-\log\left(-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2),x, algorithm="fricas")
[Out] 1/2*log(1/2*cos(x)^2 + 1/2) - log(-cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{\cos^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)**2),x)
[Out] Integral(tan(x)/(cos(x)**2 + 1), x)
```

Giac [A]
time = 0.48, size = 16, normalized size = 0.94

$$\frac{1}{2}\,\log\left(\cos\left(x\right)^{2}+1\right)-\log\left(\left|\cos\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2),x, algorithm="giac")
[Out] 1/2*log(cos(x)^2 + 1) - log(abs(cos(x)))
```

Mupad [B]
time = 2.16, size = 9, normalized size = 0.53

$$\frac{\ln\left(\tan(x)^{2}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)^2 + 1),x)
[Out] log(tan(x)^2 + 2)/2
```

3.87 $\int \sqrt{a+b\cos^2(x)}\,\tan(x)\,dx$

Optimal. Leaf size=40

$$\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\cos^2(x)}$$

```
[Out] arctanh((a+b*cos(x)^2)^(1/2)/a^(1/2))*a^(1/2)-(a+b*cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3273, 52, 65, 214}

$$\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)-\sqrt{a+b\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^2]*Tan[x],x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]] - Sqrt[a + b*Cos[x]^2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cos^2(x)}\tan(x)\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^2(x)\right)\right)\\ &= -\sqrt{a+b\cos^2(x)} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^2(x)\right)\\ &= -\sqrt{a+b\cos^2(x)} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^2(x)}\right)}{b}\\ &= \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\cos^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right) - \sqrt{a+b\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^2]*Tan[x],x]
[Out] Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]] - Sqrt[a + b*Cos[x]^2]
```

Maple [A]
time = 0.06, size = 43, normalized size = 1.08

method	result	size
derivativedivides	$-\sqrt{a+b\left(\cos^2\left(x\right)\right)}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^2\left(x\right)\right)}}{\cos\left(x\right)}\right)$	43
default	$-\sqrt{a+b\left(\cos^2\left(x\right)\right)}+\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^2\left(x\right)\right)}}{\cos\left(x\right)}\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] -(a+b*cos(x)^2)^(1/2)+a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^2)^(1/2))/cos(x
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.
time = 0.48, size = 95, normalized size = 2.38

$$\frac{1}{2}\sqrt{a}\log\left(b-\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)-1}-\frac{a}{\sin(x)-1}\right)+\frac{1}{2}\sqrt{a}\log\left(-b+\frac{\sqrt{-b\sin(x)^2+a+b}\sqrt{a}}{\sin(x)+1}+\frac{a}{\sin(x)+1}\right)-\sqrt{-b\sin(x)^2+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*log(b - sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) - 1) - a/(sin
(x) - 1)) + 1/2*sqrt(a)*log(-b + sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x)
+ 1) + a/(sin(x) + 1)) - sqrt(-b*sin(x)^2 + a + b)
```

Fricas [A]
time = 0.53, size = 90, normalized size = 2.25

$$\left[\frac{1}{2}\sqrt{a}\log\left(\frac{b\cos(x)^2+2\sqrt{b\cos(x)^2+a}\sqrt{a}+2a}{\cos(x)^2}\right)-\sqrt{b\cos(x)^2+a},-\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(x)^2+a}\sqrt{-a}}{a}\right)-\sqrt{b\cos(x)^2+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((b*cos(x)^2 + 2*sqrt(b*cos(x)^2 + a)*sqrt(a) + 2*a)/cos(x)
^2) - sqrt(b*cos(x)^2 + a), -sqrt(-a)*arctan(sqrt(b*cos(x)^2 + a)*sqrt(-a)/
a) - sqrt(b*cos(x)^2 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cos^2(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**2)*tan(x), x)
```

Giac [A]
time = 0.46, size = 38, normalized size = 0.95

$$-\frac{a\arctan\left(\frac{\sqrt{b\cos(x)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\sqrt{b\cos(x)^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)^(1/2)*tan(x),x, algorithm="giac")
[Out] -a*arctan(sqrt(b*cos(x)^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*cos(x)^2 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\,\sqrt{b\cos(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^2)^(1/2),x)
[Out] int(tan(x)*(a + b*cos(x)^2)^(1/2), x)
```

3.88 $\int \sqrt{1-\cos^2(x)}\,\tan(x)\,dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)-\sqrt{\sin^2(x)}$$

[Out] arctanh((sin(x)^2)^(1/2))-(sin(x)^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3255, 3284, 52, 65, 212}

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)-\sqrt{\sin^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Cos[x]^2]*Tan[x],x]

[Out] ArcTanh[Sqrt[Sin[x]^2]] - Sqrt[Sin[x]^2]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-\cos^2(x)}\tan(x)\,dx &= \int \sqrt{\sin^2(x)}\tan(x)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{x}}{1-x}\,dx, x, \sin^2(x)\right)\\ &= -\sqrt{\sin^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}}\,dx, x, \sin^2(x)\right)\\ &= -\sqrt{\sin^2(x)} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin^2(x)}\right)\\ &= \tanh^{-1}\left(\sqrt{\sin^2(x)}\right) - \sqrt{\sin^2(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.
time = 0.03, size = 47, normalized size = 2.35

$$-\csc(x)\sqrt{\sin^2(x)}\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]^2]*Tan[x],x]
```

```
[Out] -(Csc[x]*Sqrt[Sin[x]^2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]
] + Sin[x]))
```

Maple [A]
time = 0.34, size = 17, normalized size = 0.85

method	result

default	$-\sqrt{\frac{1}{2}-\frac{\cos(2x)}{2}}+\operatorname{arctanh}\left(\frac{2}{\sqrt{2-2\cos(2x)}}\right)$
risch	$-\frac{\sqrt{-\left(e^{2ix}-1\right)^{2}e^{-2ix}}\,e^{2ix}}{2(e^{2ix}-1)}+\frac{\sqrt{-\left(e^{2ix}-1\right)^{2}e^{-2ix}}}{2\,e^{2ix}-2}-\frac{i\sqrt{-\left(e^{2ix}-1\right)^{2}e^{-2ix}}\,e^{ix}\ln(e^{ix}-i)}{e^{2ix}-1}+\frac{i\sqrt{-(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] -(sin(x)^2)^(1/2)+arctanh(1/(sin(x)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(16) = 32$.
time = 0.48, size = 47, normalized size = 2.35

$$\frac{1}{2}\,(-1)^{2\,\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)+1}\right)+\frac{1}{2}\,(-1)^{2\,\sin(x)}\log\left(-\frac{\sin(x)}{\sin(x)-1}\right)-\sqrt{\sin(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*sin(x))*log(-sin(x)/(sin(x) + 1)) + 1/2*(-1)^(2*sin(x))*log(-si
n(x)/(sin(x) - 1)) - sqrt(sin(x)^2)
```

Fricas [A]
time = 0.40, size = 21, normalized size = 1.05

$$\frac{1}{2}\,\log\left(\sin(x)+1\right)-\frac{1}{2}\,\log\left(-\sin(x)+1\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\left(\cos(x)-1\right)\left(\cos(x)+1\right)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(-(cos(x) - 1)*(cos(x) + 1))*tan(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.
time = 0.46, size = 45, normalized size = 2.25

$$-\sqrt{-\cos\left(x\right)^2+1}+\frac{1}{2}\log\left(\sqrt{-\cos\left(x\right)^2+1}+1\right)-\frac{1}{2}\log\left(-\sqrt{-\cos\left(x\right)^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -sqrt(-cos(x)^2 + 1) + 1/2*log(sqrt(-cos(x)^2 + 1) + 1) - 1/2*log(-sqrt(-co
s(x)^2 + 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(x)\,\sqrt{1-\cos\left(x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1 - cos(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)*(1 - cos(x)^2)^(1/2), x)
```

3.89 $\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] arctanh((a+b*cos(x)^2)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3273, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]]/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^2(x)}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^2(x)}\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Cos[x]^2]/Sqrt[a]]/Sqrt[a]
```

Maple [A]
time = 0.06, size = 30, normalized size = 1.20

method	result	size
derivativedivides	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^2\left(x\right)\right)}}{\cos\left(x\right)}\right)}{\sqrt{a}}$	30
default	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^2\left(x\right)\right)}}{\cos\left(x\right)}\right)}{\sqrt{a}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^2)^(1/2))/cos(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(19) = 38.

time = 0.48, size = 81, normalized size = 3.24

$$\frac{\log\left(b - \frac{\sqrt{-b\sin\left(x\right)^{2} + a + b}\,\sqrt{a}}{\sin\left(x\right)-1} - \frac{a}{\sin\left(x\right)-1}\right)}{2\,\sqrt{a}} + \frac{\log\left(-b + \frac{\sqrt{-b\sin\left(x\right)^{2} + a + b}\,\sqrt{a}}{\sin\left(x\right)+1} + \frac{a}{\sin\left(x\right)+1}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b - sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) - 1) - a/(sin(x) - 1)
)/sqrt(a) + 1/2*log(-b + sqrt(-b*sin(x)^2 + a + b)*sqrt(a)/(sin(x) + 1) + a
/(sin(x) + 1))/sqrt(a)
```

Fricas [A]
time = 0.46, size = 67, normalized size = 2.68

$$\left[\frac{\log\left(\frac{b\cos\left(x\right)^{2}+2\,\sqrt{b\cos\left(x\right)^{2} + a}\,\sqrt{a}+2\,a}{\cos\left(x\right)^{2}}\right)}{2\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{2} + a}\,\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cos(x)^2 + 2*sqrt(b*cos(x)^2 + a)*sqrt(a) + 2*a)/cos(x)^2)/sqrt
(a), -sqrt(-a)*arctan(sqrt(b*cos(x)^2 + a)*sqrt(-a)/a)/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(x\right)}{\sqrt{a + b\cos^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**2), x)
```

Giac [A]
time = 0.49, size = 24, normalized size = 0.96

$$-\frac{\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{2} + a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b*cos(x)^2 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{b\cos(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^2)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^2)^(1/2), x)
```

3.90 $\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=11

$$\tanh^{-1}\left(\sqrt{1+\cos^2(x)}\right)$$

```
[Out] arctanh((1+cos(x)^2)^(1/2))
```

Rubi [A]

time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3273, 65, 213}

$$\tanh^{-1}\left(\sqrt{\cos^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[1 + Cos[x]^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3273

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff*x)^p/(1 - ff*x)^((m + 1)
/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\cos^2(x)}\right)$$
$$= \tanh^{-1}\left(\sqrt{1+\cos^2(x)}\right)$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\tanh^{-1}\left(\sqrt{1+\cos^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[1 + Cos[x]^2]]
```

Maple [A]
time = 0.07, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\cos^2(x)}}\right)$	10
default	$\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\cos^2(x)}}\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(1/(1+cos(x)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(9) = 18.
time = 0.48, size = 60, normalized size = 5.45

$$\frac{1}{2}\log\left(\frac{\sqrt{-\sin(x)^2+2}}{\sin(x)+1}+\frac{1}{\sin(x)+1}-1\right)+\frac{1}{2}\log\left(-\frac{\sqrt{-\sin(x)^2+2}}{\sin(x)-1}-\frac{1}{\sin(x)-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sqrt(-sin(x)^2 + 2)/(sin(x) + 1) + 1/(sin(x) + 1) - 1) + 1/2*log(-s
qrt(-sin(x)^2 + 2)/(sin(x) - 1) - 1/(sin(x) - 1) + 1)
```

Fricas [**A**]
time = 0.41, size = 16, normalized size = 1.45

$$\log\left(\frac{\sqrt{\cos\left(x\right)^{2}+1}+1}{\cos\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] log((sqrt(cos(x)^2 + 1) + 1)/cos(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{\sqrt{\cos^{2}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)**2)**(1/2),x)
[Out] Integral(tan(x)/sqrt(cos(x)**2 + 1), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(9) = 18.
time = 0.44, size = 27, normalized size = 2.45

$$\frac{1}{2}\log\left(\sqrt{\cos\left(x\right)^{2}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\cos\left(x\right)^{2}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*log(sqrt(cos(x)^2 + 1) + 1) - 1/2*log(sqrt(cos(x)^2 + 1) - 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.09

$$\int\frac{\tan(x)}{\sqrt{\cos\left(x\right)^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)^2 + 1)^(1/2),x)
[Out] int(tan(x)/(cos(x)^2 + 1)^(1/2), x)
```

3.91 $\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx$

Optimal. Leaf size=9

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)$$

```
[Out] arctanh((sin(x)^2)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3255, 3284, 65, 212}

$$\tanh^{-1}\left(\sqrt{\sin^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[1 - Cos[x]^2],x]
```

```
[Out] ArcTanh[Sqrt[Sin[x]^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3255

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
```

```
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{1-\cos^2(x)}}\,dx &= \int \frac{\tan(x)}{\sqrt{\sin^2(x)}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}}\,dx, x, \sin^2(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin^2(x)}\right) \\
&= \tanh^{-1}\left(\sqrt{\sin^2(x)}\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(9) = 18$.
time = 0.02, size = 44, normalized size = 4.89

$$\frac{\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)}{\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[1 - Cos[x]^2],x]
[Out] ((-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sin[x])/Sqrt[Sin[x]
^2]
```

Maple [A]
time = 0.22, size = 8, normalized size = 0.89

method	result	size
default	$\operatorname{arctanh}\left(\frac{2}{\sqrt{2-2\cos(2x)}}\right)$	8
risch	$-\frac{2\ln(e^{ix}-i)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}+\frac{2\ln(e^{ix}+i)\sin(x)}{\sqrt{-(e^{2ix}-1)^2 e^{-2ix}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1-cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh(1/(sin(x)^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(7) = 14$.
time = 0.48, size = 39, normalized size = 4.33

$$\frac{1}{2}\left(-1\right)^{2\sin\left(x\right)}\log\left(-\frac{\sin\left(x\right)}{\sin\left(x\right)+1}\right)+\frac{1}{2}\left(-1\right)^{2\sin\left(x\right)}\log\left(-\frac{\sin\left(x\right)}{\sin\left(x\right)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*sin(x))*log(-sin(x)/(sin(x) + 1)) + 1/2*(-1)^(2*sin(x))*log(-si
n(x)/(sin(x) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.
time = 0.38, size = 17, normalized size = 1.89

$$\frac{1}{2}\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{\sqrt{-\left(\cos\left(x\right)-1\right)\left(\cos\left(x\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)**2)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(-(cos(x) - 1)*(cos(x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(7) = 14$.
time = 0.45, size = 33, normalized size = 3.67

$$\frac{1}{2}\log\left(\sqrt{-\cos\left(x\right)^{2}+1}+1\right)-\frac{1}{2}\log\left(-\sqrt{-\cos\left(x\right)^{2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1-cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(sqrt(-cos(x)^2 + 1) + 1) - 1/2*log(-sqrt(-cos(x)^2 + 1) + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\tan(x)}{\sqrt{1-\cos(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1 - cos(x)^2)^(1/2),x)
[Out] int(tan(x)/(1 - cos(x)^2)^(1/2), x)
```

3.92 $\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx$

Optimal. Leaf size=153

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cos(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(\right.}{6a^{5/3}}$$

```
[Out] ln(cos(x))/a+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*cos(x))/a^(5/3)-1/6*b^(2/3)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*cos(x)+b^(2/3)*cos(x)^2)/a^(5/3)-1/3*ln(a+b*cos(x)^
3)/a+1/2*sec(x)^2/a-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*cos(x))/a^(1/
3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3309, 1848, 1885, 206, 31, 648, 631, 210, 642, 266}

$$-\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\cos(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\cos^2(x)\right)}{6a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}+\frac{\log(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Cos[x]^3),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Cos[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]
*a^(5/3))) + Log[Cos[x]]/a + (b^(2/3)*Log[a^(1/3) + b^(1/3)*Cos[x]])/(3*a^(
5/3)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Cos[x] + b^(2/3)*Cos[x]^2])/
(6*a^(5/3)) - Log[a + b*Cos[x]^3]/(3*a) + Sec[x]^2/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(x)}{a+b\cos^3(x)}\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x^3\left(a+bx^3\right)}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{ax}+\frac{b(-1+x^2)}{a\left(a+bx^3\right)}\right)\,dx, x, \cos(x)\right)\\
&= \frac{\log(\cos(x))}{a}+\frac{\sec^2(x)}{2a}-\frac{b\text{Subst}\left(\int \frac{-1+x^2}{a+bx^3}\,dx, x, \cos(x)\right)}{a}\\
&= \frac{\log(\cos(x))}{a}+\frac{\sec^2(x)}{2a}+\frac{b\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \cos(x)\right)}{a}-\frac{b\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \cos(x\right.}{a}\\
&= \frac{\log(\cos(x))}{a}-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \cos(x)\right)}{3a^{5/3}}+\\
&= \frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\cos^3(x)\right)}{3a}+\frac{\sec^2(x)}{2a}-\frac{b^{2/3}\text{Su}}{}\\
&= \frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cos(x)+b^{2/3}\text{c}\right.}{6a^{5/3}}\\
&= -\frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\cos(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{\log(\cos(x))}{a}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cos(x)\right)}{3a^{5/3}}-\frac{b^{2/3}\log}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.33, size = 217, normalized size = 1.42

$$\frac{6\left(\log(\cos(x))+\log\left(\sec^2\left(\frac{x}{2}\right)\right)\right)-2\text{RootSum}\left[a+b+3a\#1-3b\#1+3a\#1^2+3b\#1^2+a\#1^3-b\#1^3\&,\frac{a\log\left(-\#1+\tan^2\left(\frac{x}{2}\right)\right)+b\log\left(-\#1+\tan^2\left(\frac{x}{2}\right)\right)+2a\log\left(-\#1+\tan^2\left(\frac{x}{2}\right)\right)\#1+4b\log\left(-\#1+\tan^2\left(\frac{x}{2}\right)\right)\#1+a\log\left(-\#1+\tan^2\left(\frac{x}{2}\right)\right)\#1^2-b\log\left(-\#1+\tan^2\left(\frac{x}{2}\right)\right)\#1^2}{a-b+2a\#1+2b\#1+a\#1^2-b\#1^2}\&\right]+3\sec^2(x)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Cos[x]^3),x]
```

```
[Out] (6*(Log[Cos[x]] + Log[Sec[x/2]^2]) - 2*RootSum[a + b + 3*a*#1 - 3*b*#1 + 3*
a*#1^2 + 3*b*#1^2 + a*#1^3 - b*#1^3 & , (a*Log[-#1 + Tan[x/2]^2] + b*Log[-#
1 + Tan[x/2]^2] + 2*a*Log[-#1 + Tan[x/2]^2]*#1 + 4*b*Log[-#1 + Tan[x/2]^2]*
#1 + a*Log[-#1 + Tan[x/2]^2]*#1^2 - b*Log[-#1 + Tan[x/2]^2]*#1^2)/(a - b +
2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] + 3*Sec[x]^2)/(6*a)
```

Maple [A]
time = 0.20, size = 132, normalized size = 0.86

method	result
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2 a}+i\left(\sum_{\_R=\mathrm{RootOf}(27\_Z^3a^5-27ia^4\_Z^2-9\_Za^3+ia^2-ib^2)}\_R\ln\left(e^{2ix}+\left(\frac{6ia^2\_R}{b}+\frac{2a}{b}\right)e^{ix}+1\right)\right)+$
default	$-\frac{\left(-\frac{\ln\left(\cos(x)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(\cos^2(x)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\cos(x)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(a+b\left(\cos^3(x)\right)\right)}{3b}\right)b}{a}+\frac{\ln(\cos(x))}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*cos(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(1/b*a)^(2/3)*ln(cos(x)+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(2/3)*ln(cos(
x)^2-(1/b*a)^(1/3)*cos(x)+(1/b*a)^(2/3))-1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*cos(x)-1))+1/3/b*ln(a+b*cos(x)^3))/a*b+ln(cos
(x))/a+1/2/a/cos(x)^2
```

Maxima [A]
time = 0.48, size = 151, normalized size = 0.99

$$\frac{\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\cos(x)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2}-\frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\cos(x)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\cos(x)\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log(\cos(x))}{a}+\frac{1}{2a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b)^(1
/3) - 2*cos(x))/(a/b)^(1/3))/a^2 - 1/6*(2*(a/b)^(2/3) + 1)*log(cos(x)^2 - (
a/b)^(1/3)*cos(x) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*((a/b)^(2/3) - 1)*lo
g((a/b)^(1/3) + cos(x))/(a*(a/b)^(2/3)) + log(cos(x))/a + 1/2/(a*cos(x)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 18.86, size = 1690, normalized size = 11.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(1/3)*a*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (
a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2)*arctan(-1/8*(2*sqrt(1
/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(
1/3) + 2/a)^2*a^4 + 4*b^2*cos(x)^2 - 4*a*b*cos(x) + 2*(a^2*b*cos(x) - 2*a^3
)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
2/a) + 4*a^2)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/
a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2) + sqrt(1/3
)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
 2/a)^2*a^5 - 8*a^2*b*cos(x) + 4*a^3 + 4*(a^3*b*cos(x) - a^4)*((1/2)^(1/3)*
(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a))*sqrt((((1
/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^
2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^
(1/3) + 2/a)*a + 4)/a^2))/b^2)*cos(x)^2 - 6*sqrt(1/3)*a*sqrt((((1/2)^(1/3)*
(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*
((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/
a)*a + 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(
1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^4 + 4*b^2*cos(x)^2 - 4*
a*b*cos(x) + 2*(a^2*b*cos(x) - 2*a^3)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
 b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a) + 4*a^2)*(((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((
1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)
^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)
^(1/3) + 2/a)*a + 4)/a^2) - sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^5 - 8*a^2*b*cos(x) + 4*a^3 +
4*(a^3*b*cos(x) - a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2
 - b^2)/a^5)^(1/3) + 2/a))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/
a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2))/b^2)*cos(x)^2
 - ((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
 2/a)*a*cos(x)^2*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a
^2 - b^2)/a^5)^(1/3) + 2/a)^2*a^4 + b^2*cos(x)^2 + 2*a*b*cos(x) - (a^2*b*co
s(x) + a^3)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5
)^(1/3) + 2/a) + a^2) + 12*cos(x)^2*log(-cos(x)) + (((1/2)^(1/3)*(I*sqrt(3)
 + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) + 2/a)*a*cos(x)^2 - 6*cos(x
)^2)*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(
1/3) + 2/a)^2*a^4 + 4*b^2*cos(x)^2 - 4*a*b*cos(x) + 2*(a^2*b*cos(x) - 2*a^3
)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 - b^2)/a^5)^(1/3) +
2/a) + 4*a^2) + 6)/(a*cos(x)^2)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(x)}{a + b\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*cos(x)**3),x)
```

```
[Out] Integral(tan(x)**3/(a + b*cos(x)**3), x)
```

Giac [A]
time = 0.48, size = 143, normalized size = 0.93

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\cos(x)\right|\right)}{3\,a^2}-\frac{\log\left(\left|b\cos(x)^3+a\right|\right)}{3\,a}+\frac{\log\left(\left|\cos(x)\right|\right)}{a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\cos(x)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\cos(x)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\cos(x)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2}+\frac{1}{2\,a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)^3),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + cos(x)))/a^2 - 1/3*log(abs(b*co
s(x)^3 + a))/a + log(abs(cos(x)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3
*sqrt(3)*((-a/b)^(1/3) + 2*cos(x))/(-a/b)^(1/3))/a^2 + 1/6*(-a*b^2)^(1/3)*l
og(cos(x)^2 + (-a/b)^(1/3)*cos(x) + (-a/b)^(2/3))/a^2 + 1/2/(a*cos(x)^2)
```

Mupad [B]
time = 5.21, size = 1281, normalized size = 8.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*cos(x)^3),x)
```

```
[Out] (2*tan(x/2)^2)/(a - 2*a*tan(x/2)^2 + a*tan(x/2)^4) + log(tan(x/2)^2 - 1)/a
+ symsum(log((262144*(9*a*b^10 - b^11 - 37*a^2*b^9 + 85*a^3*b^8 - 107*a^4*b
^7 + 43*a^5*b^6 + 73*a^6*b^5 - 121*a^7*b^4 + 72*a^8*b^3 - 16*a^9*b^2))/a^6
+ root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(27*a^5*z^
3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(27*a^5*z^3 + 27*a^4*z^2 +
 9*a^3*z + a^2 - b^2, z, k)*((262144*(72*a^5*b^9 - 96*a^6*b^8 + 1428*a^7*b^
7 - 3684*a^8*b^6 + 612*a^9*b^5 + 3972*a^10*b^4 - 2112*a^11*b^3 - 192*a^12*b
^2))/a^6 + root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*(root(
27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*((262144*(5184*a^10*b^
6 - 3024*a^9*b^7 + 1728*a^11*b^5 - 6048*a^12*b^4 + 1296*a^13*b^3 + 864*a^14
*b^2))/a^6 - root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k)*((26
2144*(1296*a^10*b^7 - 3888*a^11*b^6 + 2592*a^12*b^5 + 2592*a^13*b^4 - 3888*
a^14*b^3 + 1296*a^15*b^2))/a^6 - (262144*tan(x/2)^2*(1296*a^10*b^7 - 11016*
a^11*b^6 + 27216*a^12*b^5 - 28512*a^13*b^4 + 12960*a^14*b^3 - 1944*a^15*b^2
```

```
))/a^6) + (262144*tan(x/2)^2*(4104*a^9*b^7 - 16740*a^10*b^6 + 18468*a^11*b^
5 - 1836*a^12*b^4 - 5292*a^13*b^3 + 1296*a^14*b^2))/a^6) + (262144*(288*a^7
*b^8 - 1836*a^8*b^7 - 1692*a^9*b^6 + 6084*a^10*b^5 + 108*a^11*b^4 - 4248*a^
12*b^3 + 1296*a^13*b^2))/a^6 + (262144*tan(x/2)^2*(4392*a^8*b^7 - 360*a^7*b
^8 + 3366*a^9*b^6 - 29934*a^10*b^5 + 35946*a^11*b^4 - 15354*a^12*b^3 + 1944
*a^13*b^2))/a^6) - (262144*tan(x/2)^2*(72*a^5*b^9 - 162*a^6*b^8 + 3780*a^7*
b^7 - 20160*a^8*b^6 + 30276*a^9*b^5 - 14526*a^10*b^4 + 432*a^11*b^3 + 288*a
^12*b^2))/a^6) + (262144*(68*a^4*b^9 - 436*a^5*b^8 + 903*a^6*b^7 - 55*a^7*b
^6 - 1579*a^8*b^5 + 987*a^9*b^4 + 608*a^10*b^3 - 496*a^11*b^2))/a^6 - (2621
44*tan(x/2)^2*(90*a^4*b^9 - 666*a^5*b^8 + 3753*a^6*b^7 - 5925*a^7*b^6 - 131
1*a^8*b^5 + 8919*a^9*b^4 - 5604*a^10*b^3 + 744*a^11*b^2))/a^6) - (262144*(8
*a^2*b^10 - 26*a^3*b^9 - 30*a^4*b^8 + 292*a^5*b^7 - 540*a^6*b^6 + 230*a^7*b
^5 + 402*a^8*b^4 - 496*a^9*b^3 + 160*a^10*b^2))/a^6 + (262144*tan(x/2)^2*(1
0*a^2*b^10 - 54*a^3*b^9 + 52*a^4*b^8 + 920*a^5*b^7 - 4214*a^6*b^6 + 7442*a^
7*b^5 - 6168*a^8*b^4 + 2252*a^9*b^3 - 240*a^10*b^2))/a^6) - (262144*tan(x/2
)^2*(11*a*b^10 - b^11 - 87*a^2*b^9 + 391*a^3*b^8 - 1045*a^4*b^7 + 1705*a^5*
b^6 - 1677*a^6*b^5 + 941*a^7*b^4 - 262*a^8*b^3 + 24*a^9*b^2))/a^6)*root(27*
a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + a^2 - b^2, z, k), k, 1, 3)
```

3.93 $\int \sqrt{a+b\cos^3(x)}\,\tan(x)\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+b\cos^3(x)}$$

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))*a^(1/2)-2/3*(a+b*cos(x)^3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3309, 272, 52, 65, 214}

$$\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)-\frac{2}{3}\sqrt{a+b\cos^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^3]*Tan[x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/3 - (2*Sqrt[a + b*Cos[x]^
3])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cos^3(x)}\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^3(x)\right)\right)\\
&= -\frac{2}{3}\sqrt{a+b\cos^3(x)} - \frac{1}{3}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^3(x)\right)\\
&= -\frac{2}{3}\sqrt{a+b\cos^3(x)} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^3(x)}\right)}{3b}\\
&= \frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right) - \frac{2}{3}\sqrt{a+b\cos^3(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 1.00

$$
\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right) - \frac{2}{3}\sqrt{a+b\cos^3(x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^3]*Tan[x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/3 - (2*Sqrt[a + b*Cos[x]^
3])/3
```

Maple [A]
time = 0.82, size = 34, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cos^{3}\left(x\right)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}-\frac{2\sqrt{a+b\left(\cos^{3}\left(x\right)\right)}}{3}$	34
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cos^{3}\left(x\right)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3}-\frac{2\sqrt{a+b\left(\cos^{3}\left(x\right)\right)}}{3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^3)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))*a^(1/2)-2/3*(a+b*cos(x)^3)^(1/2)
```

Maxima [A]
time = 0.48, size = 52, normalized size = 1.16

$$-\frac{1}{3}\sqrt{a}\,\log\left(\frac{\sqrt{b\cos\left(x\right)^{3}+a}-\sqrt{a}}{\sqrt{b\cos\left(x\right)^{3}+a}+\sqrt{a}}\right)-\frac{2}{3}\sqrt{b\cos\left(x\right)^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(a)*log((sqrt(b*cos(x)^3 + a) - sqrt(a))/(sqrt(b*cos(x)^3 + a) + s
qrt(a))) - 2/3*sqrt(b*cos(x)^3 + a)
```

Fricas [A]
time = 1.76, size = 123, normalized size = 2.73

$$\left[\frac{1}{6}\sqrt{a}\,\log\left(-\frac{b^{2}\cos\left(x\right)^{6}+8\,ab\cos\left(x\right)^{3}+4\left(b\cos\left(x\right)^{3}+2\,a\right)\sqrt{b\cos\left(x\right)^{3}+a}\,\sqrt{a}+8\,a^{2}}{\cos\left(x\right)^{6}}\right)-\frac{2}{3}\sqrt{b\cos\left(x\right)^{3}+a},\,-\frac{1}{3}\sqrt{-a}\,\arctan\left(\frac{2\sqrt{b\cos\left(x\right)^{3}+a}\,\sqrt{-a}}{b\cos\left(x\right)^{3}+2\,a}\right)-\frac{2}{3}\sqrt{b\cos\left(x\right)^{3}+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-(b^2*cos(x)^6 + 8*a*b*cos(x)^3 + 4*(b*cos(x)^3 + 2*a)*sqr
t(b*cos(x)^3 + a)*sqrt(a) + 8*a^2)/cos(x)^6) - 2/3*sqrt(b*cos(x)^3 + a), -1
/3*sqrt(-a)*arctan(2*sqrt(b*cos(x)^3 + a)*sqrt(-a)/(b*cos(x)^3 + 2*a)) - 2/
3*sqrt(b*cos(x)^3 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cos^{3}\left(x\right)}\,\tan\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**3)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**3)*tan(x), x)
```

Giac [A]
time = 0.44, size = 38, normalized size = 0.84

$$-\frac{2\,a\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{3}+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}-\frac{2}{3}\,\sqrt{b\cos\left(x\right)^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^3)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -2/3*a*arctan(sqrt(b*cos(x)^3 + a)/sqrt(-a))/sqrt(-a) - 2/3*sqrt(b*cos(x)^3
 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\,\sqrt{b\cos\left(x\right)^{3}+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^3)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^3)^(1/2), x)
```

3.94 $\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3309, 272, 65, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^3],x]

[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/(3*Sqrt[a])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\cos^3(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^3(x)\right)\right)\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^3(x)}\right)}{3b}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^3],x]
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.75

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cos^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$	21
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\cos^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*arctanh((a+b*cos(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.49, size = 39, normalized size = 1.39

$$-\frac{\log\left(\frac{\sqrt{b\cos\left(x\right)^{3}+a}-\sqrt{a}}{\sqrt{b\cos\left(x\right)^{3}+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*log((sqrt(b*cos(x)^3 + a) - sqrt(a))/(sqrt(b*cos(x)^3 + a) + sqrt(a)))
/sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(Expression(Complex(Integer))),fa
iled) cannot be coerced to mode SparseUnivariatePolynomial(Expression(Compl
ex(Int
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{\sqrt{a+b\cos^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**3)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**3), x)
```

Giac [A]
time = 0.48, size = 24, normalized size = 0.86

$$-\frac{2\,\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{3}+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b*cos(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{b\cos\left(x\right)^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^3)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^3)^(1/2), x)
```

3.95 $\int \sqrt{a+b\cos^4(x)}\,\tan(x)\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+b\cos^4(x)}$$

```
[Out] 1/2*arctanh((a+b*cos(x)^4)^(1/2)/a^(1/2))*a^(1/2)-1/2*(a+b*cos(x)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3308, 272, 52, 65, 214}

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+b\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^4]*Tan[x],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^4/Sqrt[a]])/2 - Sqrt[a + b*Cos[x]^4]/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+b\cos^4(x)}\,\tan(x)\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \cos^4(x)\right)\right)\\
&= -\frac{1}{2}\sqrt{a+b\cos^4(x)} - \frac{1}{4}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^4(x)\right)\\
&= -\frac{1}{2}\sqrt{a+b\cos^4(x)} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^4(x)}\right)}{2b}\\
&= \frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right) - \frac{1}{2}\sqrt{a+b\cos^4(x)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 1.00

$$\frac{1}{2}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right) - \frac{1}{2}\sqrt{a+b\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^4]*Tan[x],x]
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^4/Sqrt[a]])/2 - Sqrt[a + b*Cos[x]^4]/2
```

Maple [A]
time = 0.07, size = 44, normalized size = 0.98

method	result	size
derivativedivides	$-\frac{\sqrt{a+b\left(\cos^{4}\left(x\right)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^{4}\left(x\right)\right)}}{\cos\left(x\right)^{2}}\right)}{2}$	44
default	$-\frac{\sqrt{a+b\left(\cos^{4}\left(x\right)\right)}}{2}+\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^{4}\left(x\right)\right)}}{\cos\left(x\right)^{2}}\right)}{2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^4)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a+b*cos(x)^4)^(1/2)+1/2*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^4)^(1/2
))/cos(x)^2)
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.96

$$\frac{1}{2}\sqrt{a}\ \operatorname{arsinh}\left(-\frac{a}{\sqrt{ab}\left(\sin\left(x\right)^{2}-1\right)}\right)-\frac{1}{2}\sqrt{b\sin\left(x\right)^{4}-2\,b\sin\left(x\right)^{2}+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arcsinh(-a/(sqrt(a*b)*(sin(x)^2 - 1))) - 1/2*sqrt(b*sin(x)^4 -
2*b*sin(x)^2 + a + b)
```

Fricas [A]
time = 0.48, size = 90, normalized size = 2.00

$$\left[\frac{1}{4}\sqrt{a}\,\log\left(\frac{b\cos\left(x\right)^{4}+2\sqrt{b\cos\left(x\right)^{4}+a}\,\sqrt{a}+2\,a}{\cos\left(x\right)^{4}}\right)-\frac{1}{2}\sqrt{b\cos\left(x\right)^{4}+a}\,,-\frac{1}{2}\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{4}+a}\,\sqrt{-a}}{a}\right)-\frac{1}{2}\sqrt{b\cos\left(x\right)^{4}+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log((b*cos(x)^4 + 2*sqrt(b*cos(x)^4 + a)*sqrt(a) + 2*a)/cos(x)
^4) - 1/2*sqrt(b*cos(x)^4 + a), -1/2*sqrt(-a)*arctan(sqrt(b*cos(x)^4 + a)*s
qrt(-a)/a) - 1/2*sqrt(b*cos(x)^4 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\cos^{4}\left(x\right)}\,\tan\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**4)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**4)*tan(x), x)
```

Giac [A]
time = 0.48, size = 38, normalized size = 0.84

$$-\frac{a \arctan\left(\frac{\sqrt{b \cos(x)^4 + a}}{\sqrt{-a}}\right)}{2 \sqrt{-a}} - \frac{1}{2} \sqrt{b \cos(x)^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(sqrt(b*cos(x)^4 + a)/sqrt(-a))/sqrt(-a) - 1/2*sqrt(b*cos(x)^4
 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x) \sqrt{b \cos(x)^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^4)^(1/2), x)
```

3.96 $\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

[Out] 1/2*arctanh((a+b*cos(x)^4)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3308, 272, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^4],x]

[Out] ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]]/(2*Sqrt[a])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3308

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
```

```
((m + 1)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((a + b*ff^(n/2)*x^(n/2))^p/(1 -
 ff*x)^((m + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^4(x)\right)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^4(x)}\right)}{2b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos^4(x)}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^4],x]
[Out] ArcTanh[Sqrt[a + b*Cos[x]^4]/Sqrt[a]]/(2*Sqrt[a])
```

Maple [A]
time = 0.06, size = 31, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^4(x)\right)}}{\cos(x)^2}\right)}{2\sqrt{a}}$	31
default	$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\cos^4(x)\right)}}{\cos(x)^2}\right)}{2\sqrt{a}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*cos(x)^4)^(1/2))/cos(x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/sqrt(b*cos(x)^4 + a), x)
```

Fricas [A]
time = 0.45, size = 67, normalized size = 2.39

$$\left[\frac{\log\left(\frac{b\cos(x)^4+2\sqrt{b\cos(x)^4+a}\,\sqrt{a}+2\,a}{\cos(x)^4}\right)}{4\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos(x)^4+a}\,\sqrt{-a}}{a}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b*cos(x)^4 + 2*sqrt(b*cos(x)^4 + a)*sqrt(a) + 2*a)/cos(x)^4)/sqrt
(a), -1/2*sqrt(-a)*arctan(sqrt(b*cos(x)^4 + a)*sqrt(-a)/a)/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\sqrt{a+b\cos^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**4), x)
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.86

$$-\frac{\arctan\left(\frac{\sqrt{b\cos(x)^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*arctan(sqrt(b*cos(x)^4 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{b\cos(x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^4)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^4)^(1/2), x)
```

3.97 $\int \sqrt{a+b\cos^n(x)}\,\tan(x)\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{a+b\cos^n(x)}}{n}$$

```
[Out] 2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n-2*(a+b*cos(x)^n)^(1/2)/n
```

Rubi [A]

time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3309, 272, 52, 65, 214}

$$\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{a+b\cos^n(x)}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]^n]*Tan[x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/n - (2*Sqrt[a + b*Cos[x]^
n])/n
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cos^n(x)}\,\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx,x,\cos(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\cos^n(x)\right)}{n}\\
&= -\frac{2\sqrt{a+b\cos^n(x)}}{n}-\frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\cos^n(x)\right)}{n}\\
&= -\frac{2\sqrt{a+b\cos^n(x)}}{n}-\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cos^n(x)}\right)}{bn}\\
&= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{n}-\frac{2\sqrt{a+b\cos^n(x)}}{n}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 0.98

$$
-\frac{-2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cos^n(x)}}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]^n]*Tan[x],x]
```

```
[Out] -((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Cos[x]^n
])/n)
```

Maple [A]
time = 0.57, size = 39, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2\sqrt{a+b\left(\cos^{n}\left(x\right)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\cos^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$	39
default	$-\frac{2\sqrt{a+b\left(\cos^{n}\left(x\right)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\cos^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^n)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/n*(2*(a+b*cos(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2)
))
```

Maxima [A]
time = 0.49, size = 58, normalized size = 1.23

$$-\frac{\sqrt{a}\log\left(\frac{\sqrt{b\cos\left(x\right)^{n}+a}-\sqrt{a}}{\sqrt{b\cos\left(x\right)^{n}+a}+\sqrt{a}}\right)}{n}-\frac{2\sqrt{b\cos\left(x\right)^{n}+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*log((sqrt(b*cos(x)^n + a) - sqrt(a))/(sqrt(b*cos(x)^n + a) + sqrt(
a)))/n - 2*sqrt(b*cos(x)^n + a)/n
```

Fricas [A]
time = 0.38, size = 97, normalized size = 2.06

$$\left[\frac{\sqrt{a}\log\left(\frac{b\cos\left(x\right)^{n}+2\sqrt{b\cos\left(x\right)^{n}+a}\,\sqrt{a}+2a}{\cos\left(x\right)^{n}}\right)-2\sqrt{b\cos\left(x\right)^{n}+a}}{n},-\frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{n}+a}\,\sqrt{-a}}{a}\right)+\sqrt{b\cos\left(x\right)^{n}+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cos(x)^n + 2*sqrt(b*cos(x)^n + a)*sqrt(a) + 2*a)/cos(x)^n)
 - 2*sqrt(b*cos(x)^n + a))/n, -2*(sqrt(-a)*arctan(sqrt(b*cos(x)^n + a)*sqrt
(-a)/a) + sqrt(b*cos(x)^n + a))/n]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos^{n}(x)}\,\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**n)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x)**n)*tan(x), x)
```

Giac [A]
time = 0.49, size = 46, normalized size = 0.98

$$-\frac{2\left(\frac{ab\arctan\left(\frac{\sqrt{b\cos(x)^{n}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\cos(x)^{n}+a}\,b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^n)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -2*(a*b*arctan(sqrt(b*cos(x)^n + a)/sqrt(-a))/sqrt(-a) + sqrt(b*cos(x)^n +
a)*b)/(b*n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\,\sqrt{a + b\cos(x)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x)^n)^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x)^n)^(1/2), x)
```

3.98 $\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] 2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3309, 272, 65, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]^n],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3309

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
```

```
st[ff^(m + 1)/f, Subst[Int[x^m*((a + b*(c*ff*x)^n)^p/(1 - ff^2*x^2)^((m + 1
)/2)), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\sqrt{a+b\cos^n(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \cos(x)\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \cos^n(x)\right)}{n}\\ &= -\frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cos^n(x)}\right)}{bn}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]^n],x]
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24
default	$\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b(\cos^n(x))}}{\sqrt{a}}\right)}{n\sqrt{a}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh((a+b*cos(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [A]
time = 0.47, size = 42, normalized size = 1.45

$$-\frac{\log\left(\frac{\sqrt{b\cos\left(x\right)^{n}+a}-\sqrt{a}}{\sqrt{b\cos\left(x\right)^{n}+a}+\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] -log((sqrt(b*cos(x)^n + a) - sqrt(a))/(sqrt(b*cos(x)^n + a) + sqrt(a)))/(sq
rt(a)*n)
```

Fricas [A]
time = 0.44, size = 74, normalized size = 2.55

$$\left[\frac{\log\left(\frac{b\cos\left(x\right)^{n}+2\sqrt{b\cos\left(x\right)^{n}+a}\,\sqrt{a}+2\,a}{\cos\left(x\right)^{n}}\right)}{\sqrt{a}\,n},\ -\frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos\left(x\right)^{n}+a}\,\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*cos(x)^n + 2*sqrt(b*cos(x)^n + a)*sqrt(a) + 2*a)/cos(x)^n)/(sqrt(a)
*n), -2*sqrt(-a)*arctan(sqrt(b*cos(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{\sqrt{a+b\cos^{n}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)**n)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(a + b*cos(x)**n), x)
```

Giac [A]
time = 0.51, size = 27, normalized size = 0.93

$$-\frac{2 \arctan\left(\frac{\sqrt{b\cos\left(x\right)^{n}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b*cos(x)^n + a)/sqrt(-a))/(sqrt(-a)*n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(x)}{\sqrt{a+b\cos\left(x\right)^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)^n)^(1/2),x)
```

```
[Out] int(tan(x)/(a + b*cos(x)^n)^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```